Wetropolis: on return periods of extreme flooding events & cost-effectiveness of flood-mitigation measures

Onno Bokhove [et al.], LIFD Summer School 07-07-2025

£: EPSRC, LIFD

SoM, Leeds Institute for Fluid Dynamics, UoL, UK

Outline

In the EPSRC network "Maths Foresees", the Environment Agency (EA) posed and stated two challenges:

- **I.** How can we visualise return periods of extreme (flooding) events to a general audience in a fluid-dynamical set-up? As opposed to ....
- This challenge is posed because people (often) mistakenly think that the time between extreme events of a certain magnitude expressed by a return period is (more or less) fixed, e.g. "I am safe for $\sim$ 100years after a $1:100$year flood". (BBC interview 2019 ....)
- *Answer to challenge-I*: the visualisation of return periods in the Wetropolis flood investigator (B. et al. (2020, 2024)).

Outline-continued

In the EPSRC network "Maths Foresees", the Environment Agency posed and stated two challenges:

- . . .
- **II.** To apply mathematics to flood mitigation with tools that are comprehensible to decision-makers. As opposed to . . . .
- *Answer to challenge-II*: a graphical cost-effectiveness tool to visualise flood-mitigation scenarios.

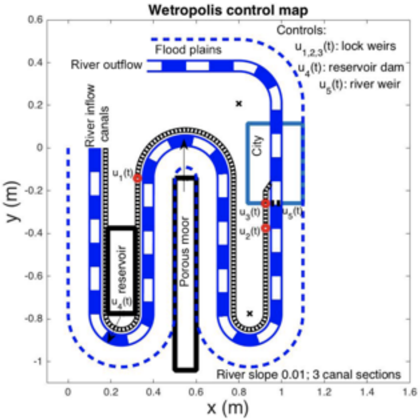
(Challenge-I) The weather machine: ingredients

The basic ideas and ingredients are the following:

- There is a conceptual river catchment with a river, a (one-sided) floodplain, a porous moor, a reservoir and downstream a city.
- Instead of 1:100 year extreme events in a $1000\mathrm{km}^2$ river catchment, say, time and spatial scales need to be reduced.
- There are only Wetropolis days (*wd*) of length T_d.
- It rains in two locations, in the moor and/or reservoir, or not: so there are 4 choices.
- It rains $(f_1, f_2, f_3, f_4)T_d$ of a day with fractions $0 < f_1 < f_2 < f_3 < f_4 < 1$: so there are ≥ 4 "daily" rain amounts possible.
- If $f_1 = 0.1$ then the rain rate during a day is a "unit" r_0.
- The river length L_r at slope 1 : 100 is $L_r \in [1, 5]\mathrm{m}$ (-).
- Remaining design unknows are therefore the day length and "unit" rainfall rate T_d, r_0.

The weather machine: map of catchment

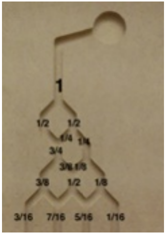
The basic ideas and ingredients on a map of "Wetropolis-I" (with LL-canal):



Inspiration for Wetropolis: Boxing Day 2015 floods of the River Aire in Leeds.

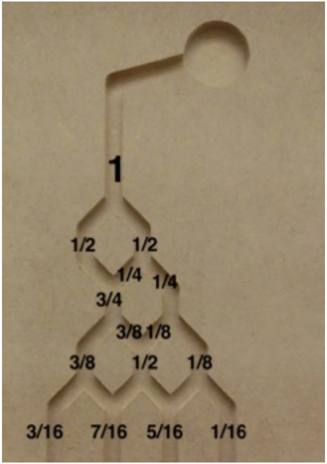
The weather machine: determine 16 outcomes

- It can rain in two locations, in the moor and/or reservoir, or not: 4 choices.
- It rains an amount $(f_1, f_2, f_3, f_4)T_d$ of a day with fractions $0 < f_1 < f_2 < f_3 < f_4 < 1$: so there are ≥ 4 "daily" rain amounts.
- Hence, there are $4 \times 4 = 16$ or rather 13 outcomes.
- (On the back of an envelope on some train ride:) Use visual draws from two discrete probability distributions each with four outcomes and a tail.
- The tail represents a rare event.



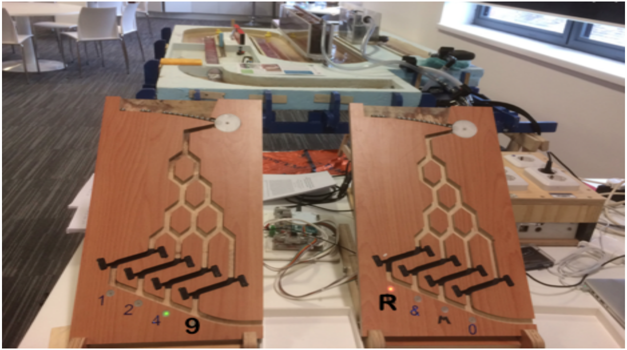
The weather machine: skewed Galton board

- Use visual draws from two discrete probability distributions each with four outcomes and a tail. Modified Galton board with 4 (or 5) rows:



The weather machine: skewed Galton boards (2016-2023)

- Use visual draws from two discrete probability distributions each with four outcomes and a tail. Two modified Galton boards each with 4 rows:



Rain duration (left: $(1, 2, 4, 9)\mathrm{s}$) and rain location (right).

Wetropolis-I weather: probability and statistics

- X, Q: probabilities p_i rainfall duration/wd versus q_j rain location:
- p_i, q_j with $i, j = 1, 2, 3, 4$ and $\sum p_i = 1, \sum q_j = 1$.
- With $p_1 = q_1 = 3/16, p_2 = q_2 = 7/16, q_3 = p_3 = 5/16, q_4 = p_4 = 1/16$:

Table: Probability matrix $P_{ij} = p_i q_j$ times 256. Rain/location (2016-2023).

	1s	2s	4s	9s
	p_1	p_2	p_3	p_4
reservoir q_1	9	21	15	3
both q_2	21	49	35	7
moor q_3	15	35	25	5
no rain q_4	3	7	5	1

The weather machine: discussion

Question: Is it unusual for a mathematician to build or propose fluid-dynamical devices and demonstrations? 몰라요:

- The inventor of the Galton board "Sir Francis Galton was a British poly-math . . . " (and mathematician).
- The innovation of Wetropolis lies in the coupling between the weather or rain machine with its skew-Galton boards and the conceptual river catchment.
- Underlying Wetropolis is a mathematical and numerical design model of PDEs, ODEs and diagnostic relations linking the equations for various components.
- Wetropolis is one member in a suite of fluid-dynamical demonstrations created with designer Wout Zweers.

... Is it unusual for a mathematician to propose fluid-dynamical devices?

That question came from a KAIST (Daejeon, Korea) member on 15-08-2024. 몰라요:

- Wetropolis is one member in a suite of my fluid-dynamical demonstrations, often based on mathematical and numerical design models.
- Note that a (PDE and ODE-based) design model aims to accommodate a design and is generally not a suitable or detailed predictive model (B. et al., HESS, 2020).
- The design model with HI-optimisation suggested a Wetropolis day length of $T_d = 10\mathrm{s} = 1\mathrm{wd}$ and unit daily rainfall rate of $r_0 \approx 0.18\mathrm{l}/\mathrm{wd}$ (B. et al., HESS, 2020).

Return period of floods: geometric distribution

- Rain amount per $T_d = 10\mathrm{s} = 1\mathrm{wd}$ determined by design: no to minor flooding for $(0, 1, 2, 4)$ & $(8, 9)$, flooding for 18 units r_0.
- Return period T_r of extreme flooding at $t_n = nT_d$ determined by geometric distribution with here $p_n = (1 - p_e)^{n-1} p_e$ where $p_e = P_{24} = q_2 p_4 = 7/256$, s.t.

$$T_r = \mathbb{E}(t_n) = \sum_{n=1}^{\infty} T_d n (1 - p_e)^{n-1} p_e = \frac{T_d}{p_e} \approx 365.7\mathrm{s} \approx 6:06\mathrm{min}.$$

- Standard deviation σ_r (thanks to Daan C. & Jason F.):

$$\sigma_r^2 = \mathbb{E}\left((t_n - \mathbb{E}(t_n))^2\right) = (1 - p_e)\frac{T_d^2}{p_e^2}$$
$$= (1 - p_e) T_r^2 \Longrightarrow \sigma_r = 36.07\mathrm{wd} = 360.7\mathrm{s} \approx 6\mathrm{min}.$$

Super- and megafloods: geometric distribution of order k

- Two consecutive "2015 Boxing Days" extreme rainfall WEP $p_e^2 = (7/256)^2$ s.t.

$$T_r^{(2)} \approx \frac{T_d}{p_e^2} = (256/7)^2 \times 10\mathrm{s} \approx 223\mathrm{min} \approx 3:43\mathrm{hr}.$$

- $T_r^{(2)}$ & $\sigma_r^{(2)}$ follow from geometric distribution of order $k = 2$ (Viveros & Balakrishnan 1993, Koutras & Eryilmaz 2017):

$$\frac{T_r^{(k)}}{T_d} = \frac{(1 - p_e^k)}{(1 - p_e)p_e^k}, \quad \frac{\sigma_r^{(k)}}{T_d} = \frac{\sqrt{1 - (2k+1)(1 - p_e)p_e^k - p_e^{2k+1}}}{(1 - p_e)p_e^k}.$$

Super- and megafloods: Wetropolis-II revisited design

- For floods on two consecutive days with old $p_e = 7/256$:

$$T_r^{(2)} = T_d \frac{(1+p_e)}{p_e^2} = 1374\mathrm{wd} = 13740\mathrm{s} = 3.8\mathrm{hr},$$
$$\sigma_r^{(2)} = T_d \frac{\sqrt{1-5(1-p_e)p_e^3 - p_e^5}}{(1-p_e)p_e^3} = 3.8\mathrm{hr}.$$

- Long waiting times suggest *redesign*, e.g. take Galton board outcome $p_e = p_2 q_2 = 49/256 \approx 1/5$ for $9\mathrm{s}$ rainfall in moor & reservoir, yielding return periods for $k = 2, 3$-day floodings:

$$T_r = 5.2\mathrm{wd} = 52\mathrm{s},\ T_r^{(2)} = 32.5\mathrm{wd} = 5:25\mathrm{min},$$
$$T_r^{(3)} = 175\mathrm{wd} = 29:11\mathrm{min}, \sigma_r^{(k)} \approx T_r^{(k)}, k = 1, 2, 3.$$

Wetropolis-II weather: revisited (2023-…)

- X, Q: probabilities p_i rainfall duration/wd versus q_j rain location.
- p_i, q_j with $i, j = 1, 2, 3, 4$ and $\sum p_i = 1, \sum q_j = 1$.
- With $p_1 = q_1 = 3/16, p_2 = q_2 = 7/16, q_3 = p_3 = 5/16, q_4 = p_4 = 1/16$:

Table: Probability matrix $P_{ij} = p_i q_j$ times 256. Current case.

	1s	7s	4s	2s
	p_1	p_2	p_3	p_4
reservoir q_1	9	21	15	3
both q_2	21	49	35	7
moor q_3	15	35	25	5
no rain q_4	3	7	5	1

Video of Wetropolis-II: visualising extreme events

Goal: visualising return period/Annual Exceedance Probability (request EA & JBA Trust). https://www.youtube.com/watch?v=yUjYfg2SfY0

Wetropolis: few remarks

- Rainfall in Wetropolis is spatio-temporal, so the occurrence/distribution of flooding events is more complicated than the imposed random rainfall distribution. TBD.
- Climate change has been implemented via a switch activating rainfall in an additional upstream extra lake/reservoir that is in sync with the random rainfall in the moor. It adds on average $\sim 20\%$ more rain to Wetropolis.
- A Galton board yields a normal distribution in the infinite-row limit. What skew-Galton-board specification would lead to other (known) skew- or extreme-value probability distributions?
- By using an LED-board with visualised "Galton-board" channels various computer-generated discrete distributions can be implemented (Robin Furze).

Visualising flood-mitigation scenarios for decision-makers

Research triggered by:

- Challenge-II stated by EA in "Maths Foresees" network 2015-2018.
- Calling a flood-evacuation of a Leeds' Crossfit-gym in the 2015 Boxing-Day floods (saving £20k, see ICS-REF2021):

How (well) can we mitigate flooding?

Flood-mitigation measures, but which ones to choose?

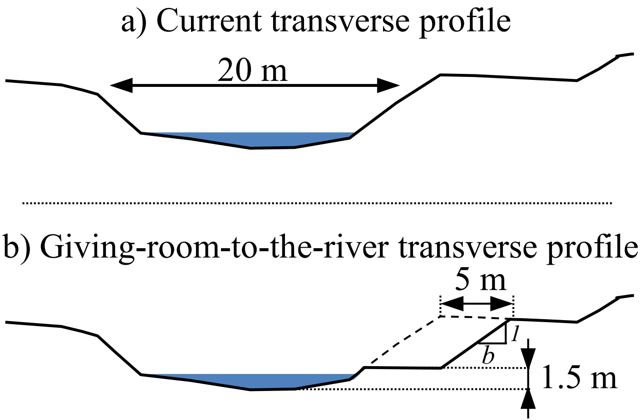
- Higher walls (HW)
- Flood-plain storage (FPS): dynamic using weirs and optimal control (underdeveloped)
- Giving-room-to-the-river (GRR)
- Natural Flood Management (NFM): tree planting, peat land, leaky dams
- Beaver colonies
- Sustainable urban drainage systems (SuDS)
- Dredging
- Resilience?

How (well) can we mitigate flooding?

Higher flood defence walls – HW ($\sim 2\mathrm{m}$ high ones in Leeds):

How (well) can we mitigate flooding?

Giving-room-to-the-river – GRR:



How (well) can we mitigate flooding?

Giving-room-to-the-river – GRR, extra channel in River Aire at *Aire River at Kirkstall The Forge* (Leeds):

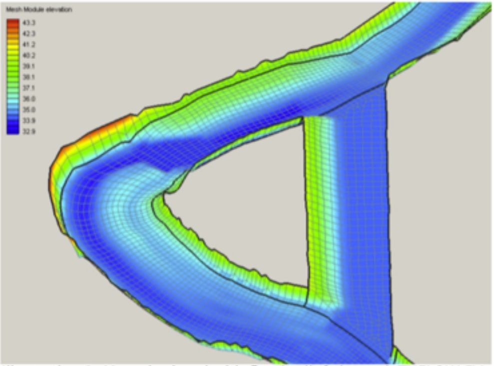


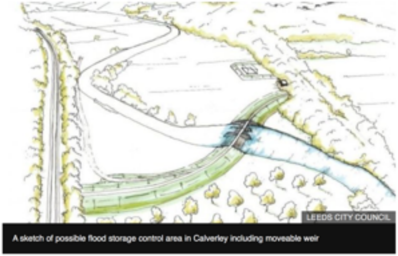
Illustration 1: Meander bend with flood relief channel, TUFLOW FV mesh

How (well) can we mitigate flooding?

Giving-room-to-the-river – GRR, extra channel in River Waal/Rhine Nijmegen (NL):

How (well) can we mitigate flooding?

Flood-plain storage –FPS & dynamic weir control:



How (well) can we mitigate flooding?

Extra storage –FPS active flooding of certain areas (Merwede, Storm Ciara, NL, $20\mathrm{Mm}^3$):

How (well) can we mitigate flooding?

Natural flood management – NFM 1300 leaky dams & trees (public engagement & co-benefits, e.g. carbon sequestration)

Central part of one of the two experimental timber bunds in the River Seven catchment

How (well) can we mitigate flooding?

Imagine your home is flooded. Lots of beaver colonies then? Extra water storage behind dams: $\sim 1100\text{m}^3 = 1.1\text{Mlitres}$ (or $1/5^{\text{th}}$).


How beavers can help stop homes from flooding

17 Feb 2020 Last updated at 11:08

Beavers can play an important role in helping to keep our homes from being flooded.

That's according to scientists at Exeter University, who have carried out a five year study of wild animals living in Devon.

They found the animals helped to [redacted] reduce pollution and boost wildlife population.

How (well) can we mitigate flooding?

SuDS –Sustainable Drainage Systems:

How (well) can we mitigate flooding?

Dredging –Wainfleet Flood Action Group (flood June 2019, 67 homes & lots of farmland flooded):

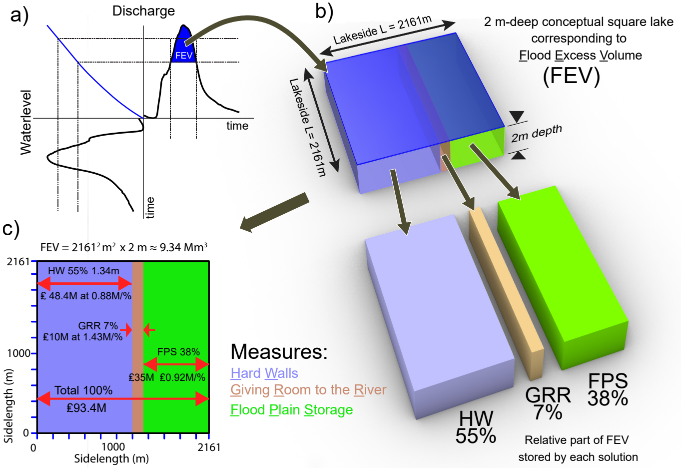
How (well) can we mitigate flooding?

Resilience: raising of new houses now mandatory in Wainfleet:

How (well) can we mitigate flooding?

Resilience: responsible flood-plain development (zero-sum or negative volume rule), Rhine valley:

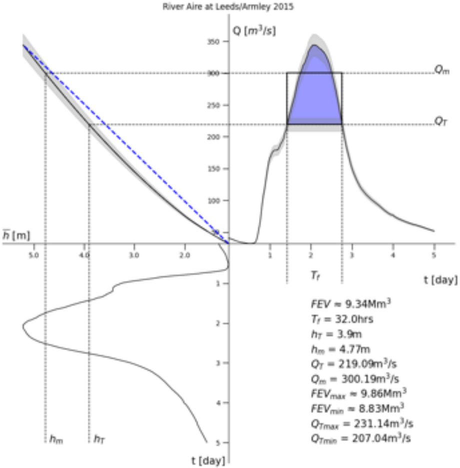
Graphical cost-effectiveness tool for flood mitigation



Graphical cost-effectiveness tool: three-panel graphs

Motivated by Boxing Day 2015 floods:
Flood-excess volume (FEV) is defined as volume of flood water one wishes to mitigate (i.e., reduce to zero) by cumulative effect of flood-mitigation measures.

Right:
River Aire gauge data of Jan. 2015 floods. **Bottom left**: $15\mathrm{min}$ water-stage time series. **Top left**: longer-time rating curve. **Top right**: resulting discharge time series.

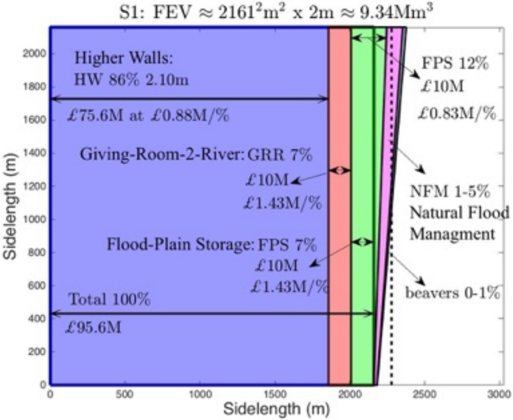


Graphical cost-effectiveness tool: square lake (1 : 200yr design flood)

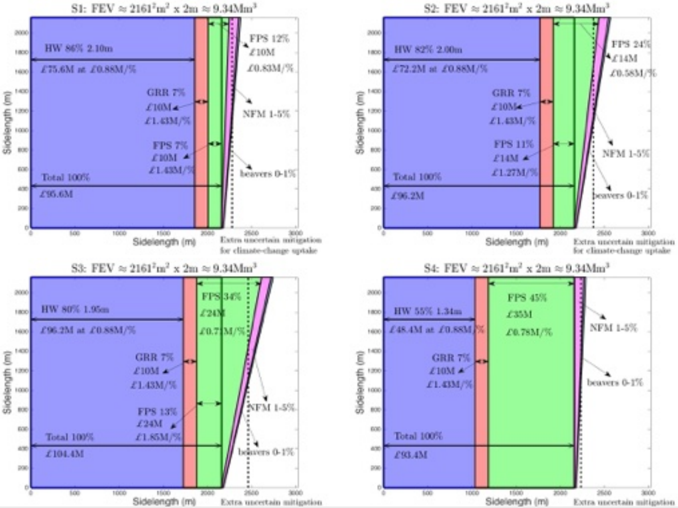
B et al. 2020 *Water*:

Scenario S1 (of 4) in a square-lake cost-effectiveness analysis of flood-alleviation plans using flood-excess volume (FEV); each mitigation measure is represented by a colour, and an overall cost analysis is displayed.

HW: higher walls,
GRR: giving-room-to-the-river,
FPS: flood plain storage,
NFM: Natural flood management,
beavers: 85 beaver colonies.



Graphical cost-effectiveness tool: square lake scenarios (1 : 200yr design flood)

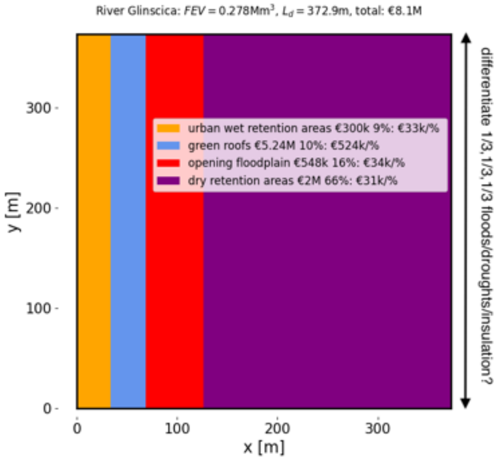


A priori FEV analysis: NBS for River Glinščica (1 : 100yrs)

Pengal et al. 2021 (EU project NAIAD):

A demonstration of participatory catchment management with stakeholders was undertaken for NBS as most suitable solution to reach these primary goals.

4 Nature Based Solutions (NBS) considered:
urban wet retention areas 9%
green roofs (floods/ droughts/ insulation) 10%
opening of flood plains 16%
dry retention areas 66%
with round-off.

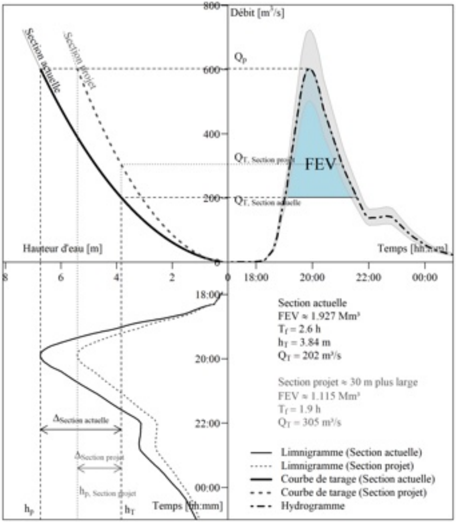


A posteriori FEV analysis: River Brague, France (1:500yrs)

Piton & Tacnet 2020 (NAIAD) after River Brague:

Based on data of hydrographs across the catchment following hydraulic simulations, FEV was calculated.

Three-panel graph of the 2015 flood of the River Brague, France, solid-line curves, as well as a GRR-modified case, dashed curves.

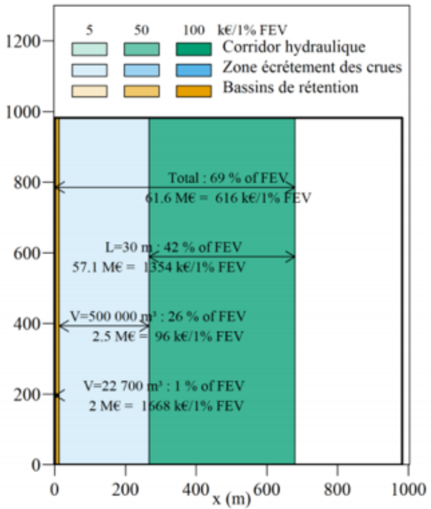


A posteriori FEV analysis: River Brague (1:500yrs)

Piton & Tacnet 2020:

Three measures cover 69% of the FEV: with **concrete basins** at 1% represented by the thin sliver, **natural retention areas** at 26% being the cheapest per percent and **GRR** at 42%.

Remaining 31% unprotected FEV requires additional measures for the worst-case design event of 1:500yrs or $AEP = 0.2\%$.



Graphical tool and its ability to find inconsistencies

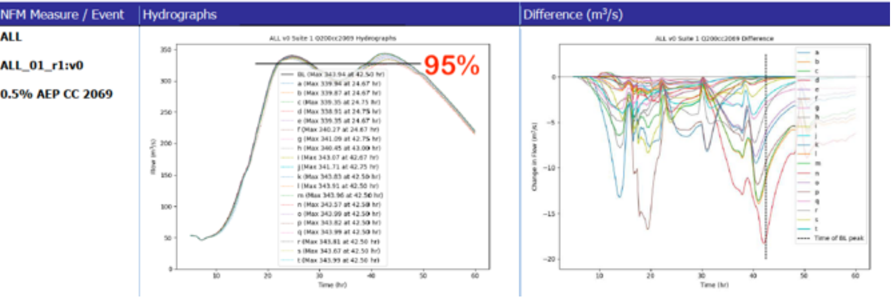
FEV and square-lake analysis-tool uncovered inconsistencies in a public [redacted] report (B et al. 2018, 2020):

- Analysis showed that efficacy of Natural Flood Management (NFM) low $[1,5]\%$ and has been (grossly) overstated;
- two vastly different flood-plain storage volumes emerge from this report leading us to define the novel concept of *available flood-storage volume*; and,
- the locations of weirs for the proposed dynamic flood-plain storage are suboptimal.

Graphical tool and its ability to find errors (02/07-2024)

FEV and square-lake analysis-tool uncovered (apparent) errors in private-public plans:

- A [redacted] Company's (RC) claim of 5% flood reduction by NFM against Climate Change effects is seemingly not seen in graphs provided as evidence, e.g., [11-06-2025: graphs in Calverley, control upstream weir and show graphs in Leeds?]:



Graphical tool and its ability to find errors (02/07-2024)

FEV and square-lake analysis-tool uncovered (apparent) errors in private-public plans:

- Response (lukewarm) by [redacted]:
  "[redacted]
- To date 20-09-2024: data sharing of relevant hydrographs has been refused by [redacted] and "*limited time for a peer review*".
- Are the (potential) investors in the flood works proposed (£4.25M needed) by RC aware of this anomaly between the claimed 5% efficacy and the (hitherto apparent lack of) evidence? Evidence provided seems to show only $\sim 2.5\%$?
- The missing $\sim 2.5\%$ was stated to come from landmanagement including soil aeration (**which is not NFM**) but corresponding evidence has hitherto been lacking.

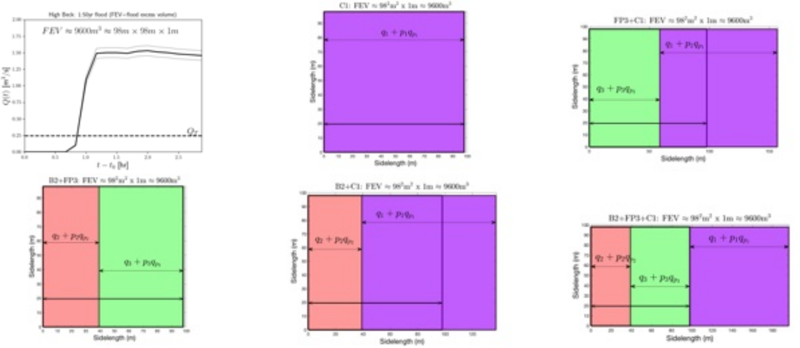
Graphical tool and its ability to find errors: remarks

Morgan and Henrion's advice (§7.8 "Uncertainty: ...", 1990) seems to apply:

- "*This means, however, that peer review should be more uniformly extended to policy focussed research and analysis than it has been in the past*".
- "*... to develop institutions and traditions that protect experts who participate in elicitation from subsequent legal or other entanglements. ... has set an excellent example by providing partial anonymity to participating experts*".
- Note that this anonymity is in apparent conflict with the UK academic and REF demands with associated funding to demonstrate impact.
- The central issue seems to be that scrutiny of public/private spending, here on flood-mitigation, by academics and especially by mathematicians, is uncommon.
- Since 07-2024, I have further flagged the (potential) issue with the EA and contractors of the RC for further (informal) investigation.
- Such a reluctance to scrutiny should be a point in a wider discussion.

High Beck flood-mitigation case study (1:10yrs)

- Square-lake plots: size & costs with flood-excess volume & mitigation measures.
- Base costs q_i, probability failure p_i, repair costs q_{p_i}, $i = 1, 2, 3$; costs $q_i + p_i q_{p_i}$.
- Combine Canal C1, bund B2, flood-plain-storage FP3 into 5 scenarios:



- Utility functions: $U_{23} = \sum_{j=1}^{5} w_j C_j$, $U_1 = \sum_{j=1}^{5}(w_j C_j - \sum_{k=1}^{N_j} \alpha_{kj} B_{jk})$ (co-benefits B_{jk}: e.g., droughts, extra CC, less pollution); if $B_{jk} = 0$: $U_1 > U_{23}$.
- If B_{jk} unknown, $U_1 = U_{23}$: appreciating benefits w. info-gap theory, B. 2025.

Discussion on visualising cost-effectiveness of flood mitigation

- FEV-analysis seemingly 0D but it captures a stretch of river, so becomes 1D.
- A priori investigation can be extended by using ensemble forecasts leading to an FEV cost-effectiveness analysis with uncertainty: see B et al. 2020 *Water* for a detailed roadmap.
- In-depth Socratic-style dialogue on critique, see B 2021 *ESREL2021*.
- In summary, the FEV cost-effectiveness approach is an essential input in the whole chain.
- It provides valuable inputs in global approaches dedicated to multifactorial analysis of flood protection measures' effectiveness.

- Note that our FEV tool is by itself and alone not a proper safety and reliability analysis approach.
- However, it is an essential input in the whole chain.

Wetropolis World: future work & proposal

How can a Wetropolis laboratory set-up and a "Numerical Wetropolis Prediction" model be used to understand:

- risk, extreme weather & flooding probability statistics –revisit spatial-temporal rainfall & change-point analysis;
- flood control –e.g., reservoirs in Wetropolis;
- data assimilation & parameter estimation –laboratory experiment as "truth run"?
- One Wetropolis World's goal: to investigate "classical" PDE & Data Assimilation "NWP" model with ML predictions.
- Proposal **EPSRC-Fellowship**$^{+}$: PDE vs. ML, info-gap theory on decision-making, 1/4 educational-version, board game, workshops.

Thank you very much for your attention ...

- B. 2025: Info-gap assessment of cost-effectiveness for flood-mitigation scenarios: Haigh Beck case study. EGU Vienna. Poster: https://obokhove.github.io/EGU2025infogapBokhove.pdf.
- Knotters, B, Lamb, Poortvliet 2024: How to cope with uncertainty monsters in flood risk management? *Cambridge Prisms: Water* **2**. https://doi.org/10.1017/wat.2024.4 (Nominated paper.)
- B 2024: High Beck fluvial flood-mitigation case study. EGU Vienna: https://obokhove.github.io/EGUBokhoveVienna2024.pdf
- B, Kelmanson, Piton, Tacnet 2024: Visualising Flood Frequency, Flood Volume and Mitigation of Extreme Events. https://obokhove.github.io/UKsuccessFEVpreprint23102023.pdf
- B 2024/2022: Wetropolis videos for general public: https://www.youtube.com/watch?v=yUjYfg2SfY0 & https://www.youtube.com/watch?v=rNgEqWdafKk
- B, Kelmanson, Hicks, Kent 2021/2022: Flood mitigation: from outreach demonstrator to a graphical cost-effectiveness diagnostic for policy makers. UK Research Excellence Framework Impact Case Study. https://results2021.ref.ac.uk/impact/submissions/1eedb5bd-8f92-4737-a6f0-1e61c997e4f0/impact
- B 2021: On communicating cost-effectiveness of flood-mitigation schemes. Angers, France. https://www.rpsonline.com.sg/proceedings/9789811820168/pdf/134.pdf
- B, Kelmanson, Piton, Tacnet 2020: A cost-effectiveness protocol for flood-mitigation plans based on . . . Boxing Day 2015 floods. *Water* **12**. https://doi.org/10.3390/w12030652
- B, Hicks, Kent, Zweers 2020: Wetropolis extreme rainfall and flood demonstrator: from mathematical design to outreach and research. *Hydrology and Earth System Sciences* **24**. https://doi.org/10.5194/hess-24-2483-2020
  Design: https://github.com/obokhove/wetropolis20162020
- B., Kent, Kelmanson, Piton, Tacnet 2019: Communicating (nature-based) solutions using flood-excess volume for three UK and French river floods. *River Research and Applications* **35**.