

개요

저는 수학하고 통계안에 나의 연구 개요를 줄 거예요:

- ▶ **웨트로폴리스 홍수 조사관** (Bokhove et al. (2020, 2024)),
- ▶ **극한 물 파도 연구**는 새로운 **파동 에너지 장치**를 영감을 했어요. (e.g., Kadomtsev & Petviashvili 1970, Benney & Luke 1964, Luke 1967, Kodama (2010, 2018), B. & Kalogirou 2016, Choi et al. (2022,2024), B. et al. (2019,2024)).

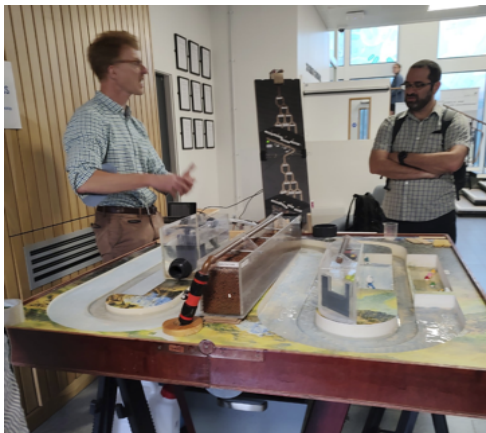
홍수에/On floods: 웨트로폴리스 홍수 조사관



Inspiration for Wetropolis: **Boxing Day 27-12-2015 floods** of the River Aire in Leeds

여기는 웨트로폴리스예요: 극한 환경 이벤트에 시각하기

Goal: 반환 기간/연간 초과 확률 시각하기 (요청으로, EA & JBA Trust). <https://www.youtube.com/watch?v=rNgEqWdafKk>



도전/Challenge

- ▶ People often classify extreme events by saying: that was an **extreme (rainfall or flooding) event** with a magnitude corresponding to a return period of 1:100 years.
- ▶ Then they can be confused to think that such an event does not occur for (approximately) another 100 years.
- ▶ For rainfall, that might mean that this extreme rainfall of (say) 50mm/day had a 1:100 year return period.
- ▶ **Wetropolis aims to visualise return periods** for the general public.
- ▶ To avoid long (say 100 year) average waiting times, **we need to shrink time** and **reduce the rainfall to a small-scale set-up** such that viewers can experience return periods of extreme events in minutes.
- ▶ **Challenge: what time duration should we have and what is a suitable unit amount** $10r_0$ per wd in a small-scale set-up?

Legacy: 우리의 홍수기 연구는 다음은 유산이 있어요

- ▶ **Leeds' public 2017 Flood-Alleviation Scheme II 분석는:** led to graphical flood-mitigation cost-effectiveness tool, laying bare inconsistencies in FASII.
- ▶ **웨트로폴리스는 비용 효율성 도구 영감을 했어요:** used in flood cases France & Slovenia
- ▶ 도구는 shows that **efficacy of Natural Flood Management** small to minute, e.g. beaver dams, contrary to statements on NFM & beavers by Environment Agency, council & media.
- ▶ **REF Impact case study ICS 2021 봐:** Wetropolis & flood-mitigation effectiveness tool for decision-makers. <https://results2021.ref.ac.uk/impact/submissions/1eedb5bd-8f92-4737-a6f0-1e61c997e4f0/impact>



우리는 어떻게 홍수를 완화시킬 수 있어요?

어느 우리의 홍수 완화시키 선택할 수 있어요?

- ▶ Higher walls (HW)
- ▶ Flood-plain storage (FPS): dynamic using weirs and optimal control (underdeveloped)
- ▶ Giving-room-to-the-river (GRR)
- ▶ Natural Flood Management (N_FM): tree planting, peat land, leaky dams
- ▶ Beaver colonies
- ▶ Sustainable urban drainage systems (SUDS)
- ▶ Dredging
- ▶ Resilience?

우리는 어떻게 홍수를 완화시킬 수 있어요?

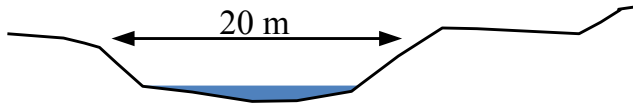
Higher flood defence walls – HW (2m high proposed in Leeds):



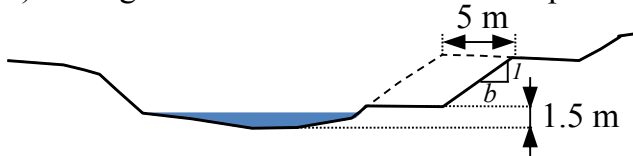
우리는 어떻게 홍수를 완화시킬 수 있어요?

Giving-room-to-the-river – GRR:

a) Current transverse profile



b) Giving-room-to-the-river transverse profile



우리는 어떻게 홍수를 완화시킬 수 있어요?

Giving-room-to-the-river – GRR, extra channel in River Aire at *Aire River at Kirkstall The Forge* (Leeds):

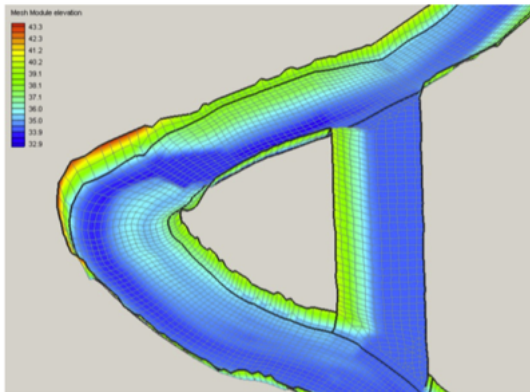


Illustration 1: Meander bend with flood relief channel, TUFLOW FV mesh

우리는 어떻게 홍수를 완화시킬 수 있어요?

Giving-room-to-the-river – GRR, extra channel in River Waal/Rhine Nijmegen (NL):



우리는 어떻게 홍수를 완화시킬 수 있어요?

Flood-plain storage –FPS & dynamic weir control:



우리는 어떻게 홍수를 완화시킬 수 있어요?

Extra storage –FPS active flooding of certain areas (Merwede, Storm Ciara, NL, 20Mm³):



우리는 어떻게 홍수를 완화시킬 수 있어요?

Natural flood management – NFM 1300 leaky dams & trees (public engagement & co-benefits, e.g. carbon sequestration)



Central part of one of the two experimental timber bunds in the River Seven catchment

우리는 어떻게 홍수를 완화시킬 수 있어요? 비버는 안 효과적여요!

Imagine your home is flooded. Lots of **beaver colonies** then? Extra water storage behind dams: $\sim 1100\text{m}^3 = 1.1\text{Mlitres}$ (or $1/5^{\text{th}}$).

How beavers can help stop homes from flooding

© 17 Feb 2020 Last updated at 11:08



Beavers can play an important role in helping to keep our homes from being flooded.

That's according to scientists at Exeter University, who have carried out a five year study of wild animals living in Devon.

우리는 어떻게 홍수를 완화시킬 수 있어요?

Dredging –Wainfleet Flood Action Group (flood June 2019, 67 homes & lots of farmland flooded):



우리는 어떻게 홍수를 완화시킬 수 있어요?

Resilience: raising of new houses now mandatory in Wainfleet:

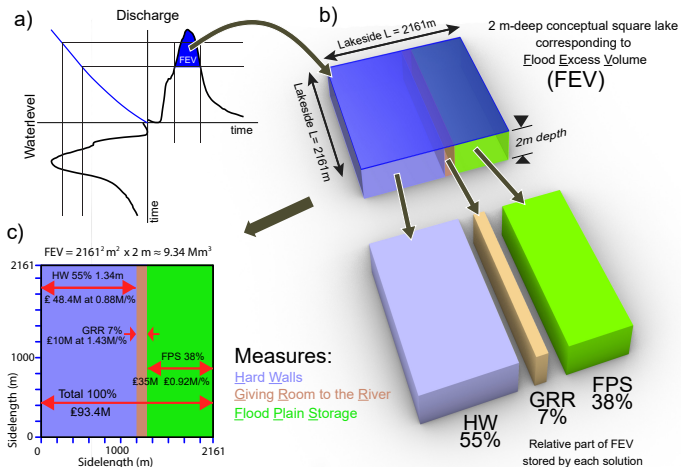


우리는 어떻게 홍수를 완화시킬 수 있어요?

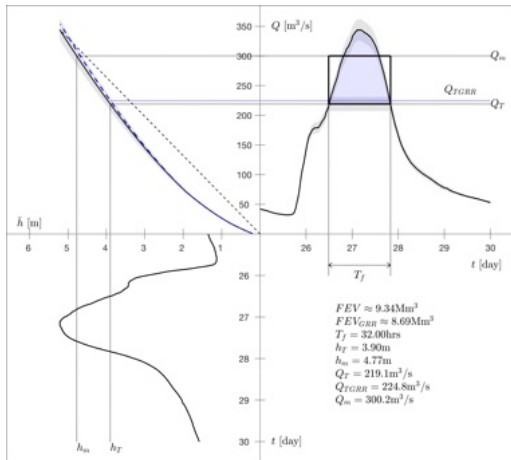
Resilience: responsible flood-plain development (**zero-sum or negative volume rule**), Rhine valley:



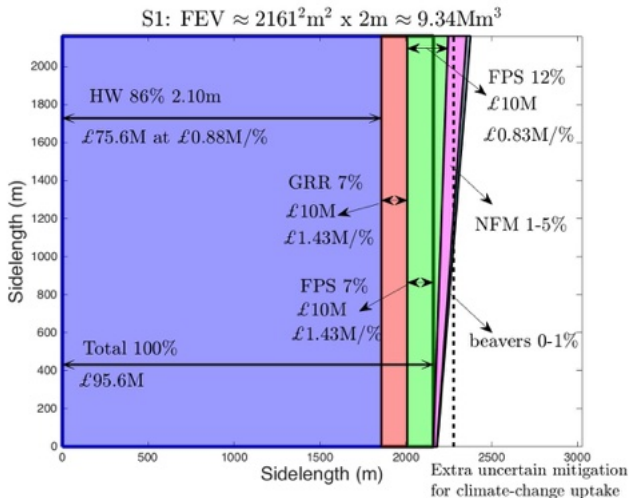
그래픽 비용 효율성 도구



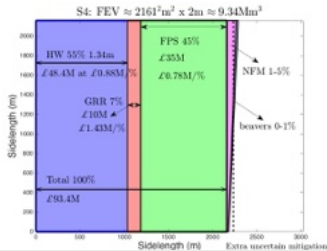
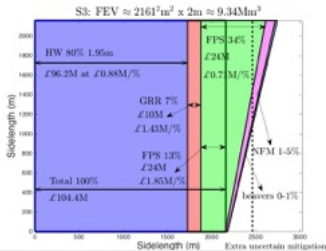
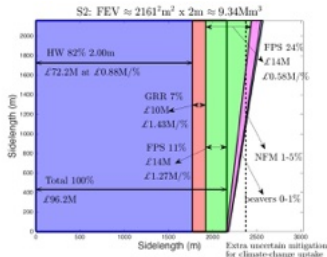
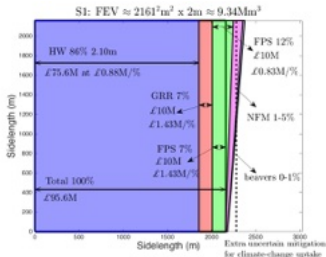
그래픽 비용 효율성 도구: 그래프와 세 사분면



그래픽 비용 효율성 도구: 사각 호수

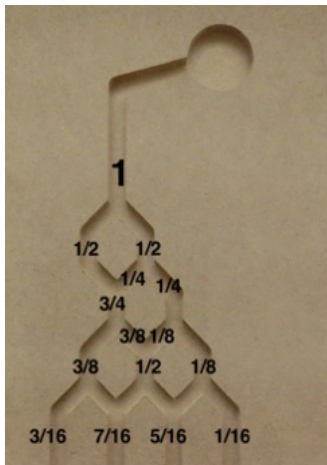


그래픽 비용 효율성 도구: 프레임워크 하고 사각 호수 (1:200yrs)



웨트로폴리스의 날씨: 왜곡한 갈튼 보드

Ball falls through, peak chance at $7/16$ & “rare” event at $1/16$:



웨트로폴리스의-I 날씨: 왜곡한 갈튼 보드 두 개

갈튼 보드 두 개: 비의 위치경우 보드 한 개하고 비의 지속
시간경우 보드 한 개:



웨트로폴리스의-1 날씨: 확률과 통계

- ▶ X, Q : probabilities p_i rainfall duration/wd versus q_j rain location:
- ▶ p_i, q_j with $i, j = 1, 2, 3, 4$ and $\sum p_i = 1, \sum q_j = 1$.
- ▶ For old case, $p_1 = q_1 = 3/16, p_2 = q_2 = 7/16, p_3 = p_3 = 5/16, q_4 = p_4 = 1/16$:

Table: Probability matrix $P_{ij} = p_i q_j$ times 256.

	1s	2s	4s	9s
	p_1	p_2	p_3	p_4
reservoir q_1	9	21	15	3
both q_2	21	49	35	7
moor q_3	15	35	25	5
no rain q_4	3	7	5	1

홍수의 반환 기간: 기하학적 분포

- ▶ Rain amount per $T_d = 10\text{s} = 1\text{wd}$ determined by **tuning**: no to minor flooding for (1, 2, 4) & (8, 9), **flooding** for 18 units r_0 .
- ▶ **Return period** T_r of extreme flooding at $t_n = nT_d$ determined by geometric distribution with here $p_n = (1 - p_e)^{n-1} p_e$ where $p_e = P_{24} = q_2 p_4 = 7/256$, s.t.

$$T_r = \mathbb{E}(t_n) = \sum_{n=1}^{\infty} T_d n (1 - p_e)^{n-1} p_e = \frac{T_d}{p_e} \approx 365.7\text{s} \approx \mathbf{6 : 06\text{min.}}$$

- ▶ Standard deviation σ_r (thanks to Daan C & Jason F):

$$\begin{aligned} \sigma_r^2 &= \mathbb{E}((t_n - \mathbb{E}(t_n))^2) = (1 - p_e) \frac{T_d^2}{p_e^2} \\ &= (1 - p_e) T_d^2 \implies \sigma_r = 36.07\text{wd} = 360.7\text{s} \approx 6\text{min.} \end{aligned}$$

슈퍼 홍수와 대규모 홍수: 차수 k 의 기하학적 분포

- ▶ Two consecutive “2015 Boxing Days” of extreme rainfall
WEP $p_e^2 = (7/256)^2$ s.t.

$$T_r^{(2)} \approx \frac{T_d}{p_e^2} = (256/7)^2 \times 10\text{s} \approx 223\text{min} \approx 3 : 43\text{hr.}$$

Movie “Wetropolis Boxing Day flood” on <https://github.com/obokhove/wetropolis20162020>

- ▶ $T_r^{(k)}$ & $\sigma_r^{(k)}$ follow from **geometric distribution of order $k = 2$**
(Viveros & Balakrishnan 1993, Koutras & Eryilmaz 2017):

$$\frac{T_r^{(k)}}{T_d} = \frac{(1 - p_e^k)}{(1 - p_e)p_e^k}, \quad \frac{\sigma_r^{(k)}}{T_d} = \frac{\sqrt{1 - (2k + 1)(1 - p_e)p_e^k - p_e^{2k+1}}}{(1 - p_e)p_e^k}.$$

슈퍼 홍수와 대규모 홍수: 웨트로폴리스-II의 재검토된 디자인

- ▶ For floods on two consecutive days with old $p_e = 7/256$:

$$T_r^{(2)} = T_d \frac{(1 + p_e)}{p_e^2} = 1374 \text{wd} = 13740 \text{s} = 3.8 \text{hr},$$

$$\sigma_r^{(2)} = T_d \frac{\sqrt{1 - 5(1 - p_e)p_e^3 - p_e^5}}{(1 - p_e)p_e^3} = 3.8 \text{hr}.$$

- ▶ Long waiting times suggest *redesign*, e.g. take Galton board outcome $p_e = p_2 q_2 = 49/256 \approx 1/5$ for 9s rainfall in moor & reservoir, yielding **return periods for $k = 2, 3$ -day floodings**:

$$T_r = 5.2 \text{wd} = 52 \text{s}, T_r^{(2)} = 32.5 \text{wd} = 5 : 25 \text{min},$$

$$T_r^{(3)} = 175 \text{wd} = 29 : 11 \text{min}, \sigma_r^{(k)} \approx T_r^{(k)}, k = 1, 2, 3.$$

다시 찾은 웨트로폴리스-II의 재검토된 날씨

- ▶ X, Q : probabilities p_i rainfall duration/wd versus q_j rain location: <https://www.youtube.com/watch?v=g8znktYpxvY>
- ▶ p_i, q_j with $i, j = 1, 2, 3, 4$ and $\sum p_i = 1, \sum q_j = 1$.
- ▶ For current case, $p_1 = q_1 = 3/16, p_2 = q_2 = 7/16, q_3 = p_3 = 5/16, q_4 = p_4 = 1/16$:

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웨트로폴리스 홍수 조사관하고: 미래의 일과 제안하려면

How can a **Wetropolis laboratory set-up and a Numerical Wetropolis Prediction** model be used to understand:

- ▶ risk, extreme weather & flooding probability statistics –revisit **spatial-temporal rainfall** & change-point analysis;
- ▶ rare-event simulations (for events of “intermediate rarity”);
- ▶ flood control –e.g., reservoirs in Wetropolis;
- ▶ data assimilation & parameter estimation –experiment as “truth run”; test (limits of) **machine learning**;
- ▶ Wetropolis World’s **goal**: investigate “classical” PDE & Data Assimilation prediction model with ML predictions.
- ▶ **Proposal** EPSRC-F⁺ (Korea?): PDE/ML, info-gap theory on decision-making, 1/4 educational-version, board game, workshops.

파도에: 개요

파도에/On waves:

- ▶ **Introduction** via movie bore-soliton-splash.
- ▶ **Eight-to-ninefold** amplification three-soliton interactions.
- ▶ Sketch & movie wave-energy device.
- ▶ **Achievement**: model device with one grand variational principle.
- ▶ Preliminary optimisations.
- ▶ Conclusions.

파도에: 모델로 하기 극한의 높은 물 파도

- ▶ Origin 2010 *bore-soliton-splash*:
- ▶ **Definition** rogue/extreme waves:
 $Al = H_r/H_s > 2$.
- ▶ Wave-energy device inspired by this amplification in a contraction.



파동 에너지 장치의 전반적인 수학적 변분 원리

Equations of motion follow from 수학적 변분 원리 (red=waves, blue=buoy, green=EM-generator, coupling):

$$0 = \delta \int_0^T \int_0^{L_x} \int_{R(t)}^{l_y(x)} \int_0^h -(\partial_t \phi + \frac{1}{2} |\nabla \phi|^2) dz - gh(\frac{1}{2}h - H_0) - \frac{1}{2\gamma} \left(F_+ (\gamma(h - h_b) - \lambda)^2 - \lambda^2 \right) dy dx$$

$$MW\dot{Z} - \frac{1}{2}MW^2 - MgZ + (L_i I - \underline{K(Z)})\dot{Q} - \frac{1}{2}L_i I^2 dt \quad (1)$$

velocity $u = \nabla \phi(x, y, z, t)$, depth $h(x, y, t)$, rest depth H_0 , buoy $h_b(Z, x) = Z - K - \tan \theta(L_y - x)$, coupling function $\gamma_m G(Z) = K'(Z)$, buoy mass M , keel height K , buoy coordinate $Z(t)$, buoy velocity $W(t) = \dot{Z}$, charge $Q(t)$, current $I(t) = \dot{Q}$.

모델로 하기: 편미분 방정식/PDEs

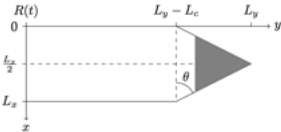
- ▶ Potential-flow water-wave dynamics (Laplace equation in interior, kinematic & Bernoulli equations at free surface):

$$\delta\phi : \nabla^2\phi = 0 \quad \text{in } \Omega$$

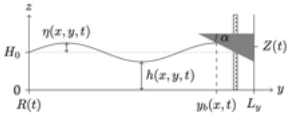
$$(\delta\phi)|_{z=h} : \partial_t h + \nabla\phi \cdot \nabla h = \phi_z \quad \text{at } z = h$$

$$\delta h : \partial_t\phi + \frac{1}{2}|\nabla\phi|^2 + g(z - H_0) - \lambda = 0 \quad \text{at } z = h.$$

- ▶ Coupled **elliptic Laplace equation** to **hyperbolic free-surface equations**, plus a (Lagrange) **multiplier** λ .



(b) Top view of the tank and buoy, outlining the tank's dimensions and how the buoy fits the shape of the contraction.



(c) Side view at time t , with the buoy constrained to move vertically.

수학적 모델로 하기: 부등식 제약 한 개 상미분 방정식

- ▶ Karush-Kuhn-Tucker inequality conditions satisfied at every space-time x , t -position are:

$$\delta\lambda : \lambda = -[\gamma(h - h_b) - \lambda]_+ = -F_+(\gamma(h - h_b) - \lambda)$$

$$\implies \underline{h(x, t) - h_b(Z, x) \leq 0, \lambda \leq 0, \lambda(h - h_b) = 0.}$$

- ▶ Add **resistance** R_i, R_c & **Shockley load** $V_s(|I|)$ to submodel:

$$\delta W : \dot{Z} = W,$$

$$\delta Z : M\dot{W} = - Mg - \underline{\gamma_m G(Z) I} - \int_0^{L_x} \int_0^{l_y(x)} \lambda dy dx$$

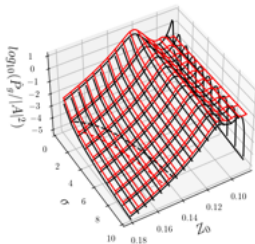
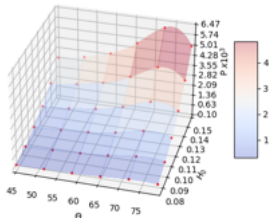
$$\delta I : \dot{Q} = I,$$

$$\delta Q : L_i \dot{I} = \underline{\gamma_m G(Z) \dot{Z}} - (R_i + R_c) I - \frac{I}{|I|} V_s(|I|).$$

파동 에너지 장치의 최적화

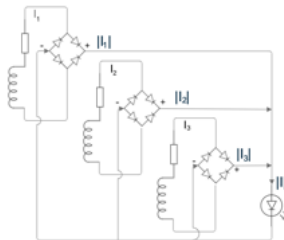
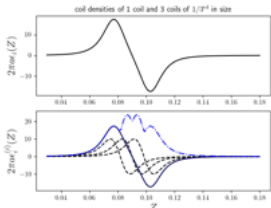
Surrogate modelling geometry angle/rest depth θ_c-H_0 &
1-coil (**black**) vs. 3-coil (**red**) power P_g (B. et al IEEE2024):

RBF approximation of Power Output beta=2.0 and n=36



새로운 파동 에너지 장치: 미래의 일

- ▶ **Brief overview given** of wave-energy device, based on extreme-wave amplification in a contraction. Showcased modelling with VPs & geometric numerical integrators.
- ▶ **미래의 일**: optimisation, real-time control using Pontryagin's principle, surrogate modelling & ML.
- ▶ Experiment soliton interactions: <https://www.youtube.com/watch?v=cBhr0DnVc1U>
- ▶ **Laboratory realisation under development**, for testing of submodels & wave-to-wire models.



감사합니다!

- ▶ Knotters, B, Lamb, Poortvliet 2024: How to cope with uncertainty monsters in flood risk management? *Water Prisms*. <https://doi.org/10.1017/wat.2024.4> (Nominated paper.)
- ▶ B, Kelmanson, Piton, Tacnet 2024: Visualising Flood Frequency, Flood Volume and Mitigation of Extreme Events. <https://obokhove.github.io/UKsuccessFEVpreprint23102023.pdf>
<https://www.youtube.com/watch?v=g8znktYpxvY>
- ▶ B, Kelmanson, Hicks, Kent: 2022: Flood mitigation: from outreach demonstrator to a graphical cost-effectiveness diagnostic for policy makers. UK Research Excellence Framework Impact Case Study. <https://results2021.ref.ac.uk/impact/submissions/1eedb5bd-8f92-4737-a6f0-1e61c997e4f0/impact>
- ▶ B, Hicks, Kent, Zweers 2020: Wetropolis extreme rainfall and flood demonstrator: from mathematical design to outreach and research. *Hydrology and Earth System Sciences* **24**.
[/doi.org/10.5194/hess-24-2483-2020](https://doi.org/10.5194/hess-24-2483-2020)
- ▶ B., Kalogirou, Zweers 2019: From bore-soliton-splash to a new wave-to-wire wave-energy model. *Water Waves* **1** 10.1007/s42286-019-00022-9 Bore-soliton-splash:
<https://www.youtube.com/watch?v=YSXsXNX4zW0&list=FL6mc7mUa6M4Bo2VkD970urw>
- ▶ Choi, Kalogirou, Lu, B., Kelmanson 2024: A study of extreme water waves using a hierarchy of models based on potential-flow theory. *Water Waves* <https://doi.org/10.1007/s42286-024-00084-4>
- ▶ B., Bolton, H. Thompson, Geometric power optimisation of a rogue-wave energy device in a (breakwater) contraction. 8th IEEE Conference on Control Technology & Applications (CCTA) (2024) 6 pp. Preprint <https://eartharxiv.org/repository/view/7260/>



세 솔리톤의 상호작용의 차원 결과

Crossing seas (8 domains combined) <https://www.youtube.com/watch?v=EGhpQ7BM2jA>

