Extreme events in Wetropolis flood investigator & dynamics of extreme water waves

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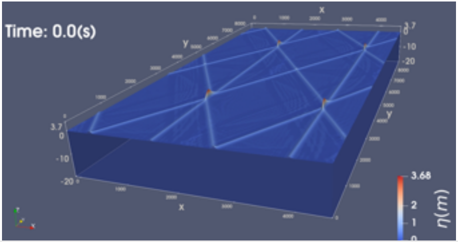


Outline

I will give an overview of our work on the mathematics and statistics of:

- Wetropolis flood investigator (B et al. (2020, 2024)),
- a novel wave-energy device based on extreme and extremely-high water waves (e.g., Kadomtsev & Petviashvilli 1970, Benney & Luke 1964, Luke 1967, Kodama (2010, 2018), B. and Kalogirou 2016, Choi et al. (2022,2024), B. et al (2019,IEEE2024)).

On floods: Wetropolis flood investigator

Inspiration for Wetropolis: Boxing Day 2015 floods of the River Aire in Leeds

Tour of Wetropolis: visualising extreme events

Goal: visualising return period/Annual Exceedance Probability (request EA & JBA Trust). https://www.youtube.com/watch?v=rNgEqWdafKk

Legacy of our work on flooding

- **Analysis Leeds' public 2017 Flood-Alleviation Scheme II:** led to graphical flood-mitigation cost-effectiveness tool, laying bare inconsistencies in FASII.
- **Wetropolis inspired cost-effectiveness tool:** used in flood cases France & Slovenia
- Tool shows that **efficacy of Natural Flood Management** small to minute, e.g. beaver dams, somewhat contrary to overstated promotion of NFM & beavers by Environment Agency, council & media.
- **See REF Impact case study ICS 2021:** Wetropolis & flood-mitigation effectiveness tool for decision-makers. `https://results2021.ref.ac.uk/impact/submissions/1eedb5bd-8f92-4737-a6f0-1e61c997e4f0/impact`

How (well) can we mitigate flooding?

Flood-mitigation measures, but which ones to choose?

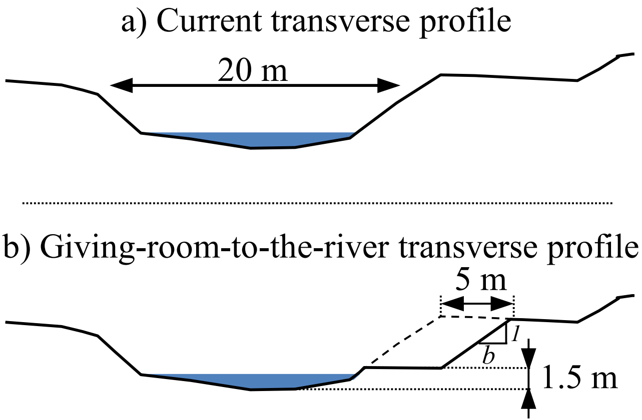
- Higher walls (HW)
- Flood-plain storage (FPS): dynamic using weirs and optimal control (underdeveloped)
- Giving-room-to-the-river (GRR)
- Natural Flood Management (NFM): tree planting, peat land, leaky dams
- Beaver colonies
- Sustainable urban drainage systems (SUDS)
- Dredging
- Resilience?

How (well) can we mitigate flooding?

Higher flood defence walls – HW ($2\mathrm{m}$ high proposed in Leeds):

How (well) can we mitigate flooding?

Giving-room-to-the-river – GRR:



How (well) can we mitigate flooding?

Giving-room-to-the-river – GRR, extra channel in River Aire at *Aire River at Kirkstall The Forge* (Leeds):

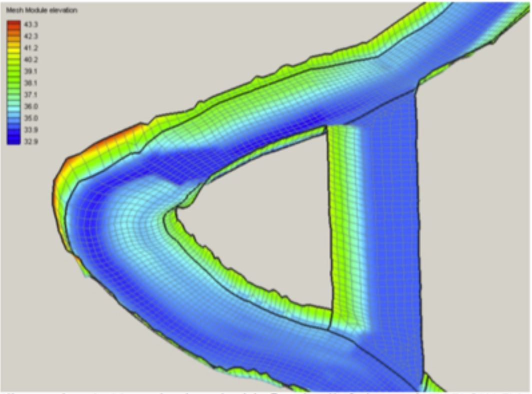


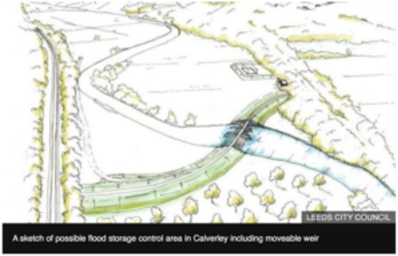
Illustration 1: Meander bend with flood relief channel, TUFLOW FV mesh

How (well) can we mitigate flooding?

Giving-room-to-the-river – GRR, extra channel in River Waal/Rhine Nijmegen (NL):

How (well) can we mitigate flooding?

Flood-plain storage –FPS & dynamic weir control:



How (well) can we mitigate flooding?

Extra storage –FPS active flooding of certain areas (Merwede, Storm Ciara, NL, $20\mathrm{Mm}^3$):

How (well) can we mitigate flooding?

Natural flood management – NFM 1300 leaky dams & trees (public engagement & co-benefits, e.g. carbon sequestration)

Central part of one of the two experimental timber bunds in the River Seven catchment

How (well) can we mitigate flooding? Beavers nonsense!

Imagine your home is flooded. Lots of beaver colonies then? Extra water storage behind dams: $\sim 1100\mathrm{m}^3 = 1.1$Mlitres (or $1/5^{\mathrm{th}}$).

How beavers can help stop homes from flooding

17 Feb 2020 Last updated at 11:08

Beavers can play an important role in helping to keep our homes from being flooded.

That's according to scientists at Exeter University, who have carried out a five year study of wild animals living in Devon.

How (well) can we mitigate flooding?

Dredging –Wainfleet Flood Action Group (flood June 2019, 67 homes & lots of farmland flooded):

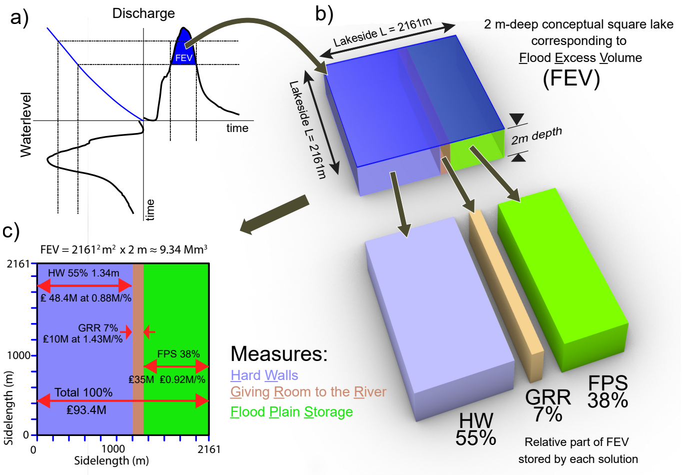
How (well) can we mitigate flooding?

Resilience: raising of new houses now mandatory in Wainfleet:

How (well) can we mitigate flooding?

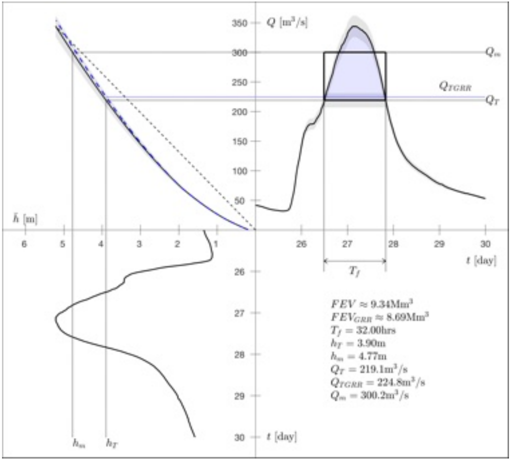
Resilience: responsible flood-plain development (zero-sum or negative volume rule), Rhine valley:

Graphical cost-effectiveness tool for flood mitigation

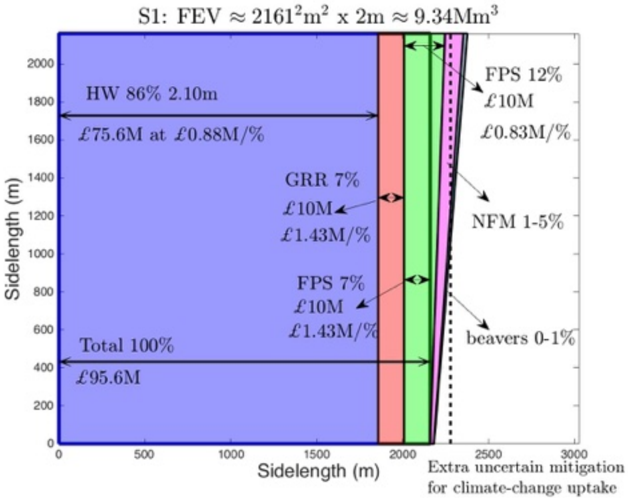


Measures:
Hard Walls
Giving Room to the River
Flood Plain Storage

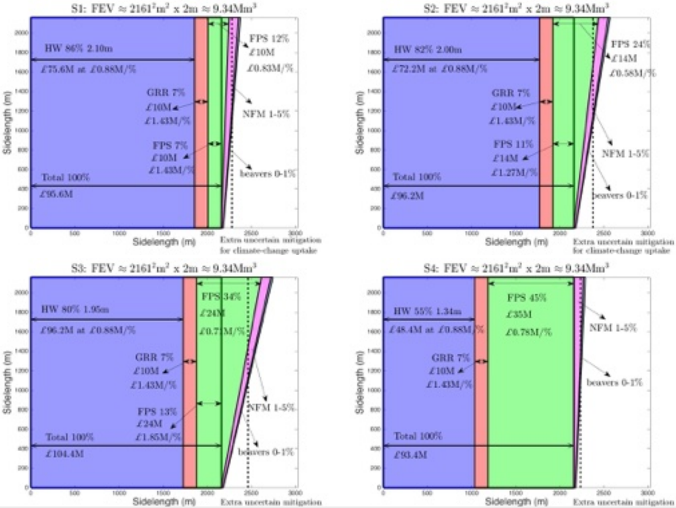
Graphical cost-effectiveness tool: three-panel graphs



Graphical cost-effectiveness tool: square lake (1 : 200yr design flood)

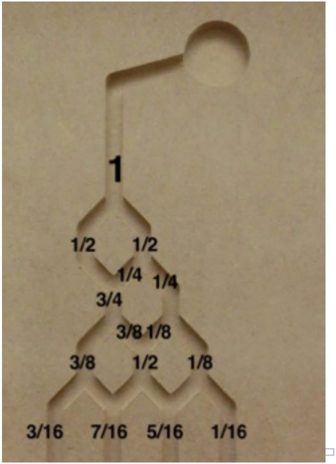


Graphical cost-effectiveness tool: square lake scenarios (1 : 200yr design flood)



Weather in Wetropolis: skew Galton boards

Ball falls through, peak chance at 7/16 & "rare" event at 1/16:



Weather in Wetropolis-I: two skew Galton boards

Two Galtonboards, one rain duration & one for rain location:

Wetropolis-I's weather: probability and statistics

- X, Q: probabilities p_i rainfall duration/wd versus q_j rain location:
- p_i, q_j with $i, j = 1, 2, 3, 4$ and $\sum p_i = 1, \sum q_j = 1$.
- For old case, $p_1 = q_1 = 3/16, p_2 = q_2 = 7/16, q_3 = p_3 = 5/16, q_4 = p_4 = 1/16$:

Table: Probability matrix $P_{ij} = p_i q_j$ times 256.

	1s	2s	4s	9s
	p_1	p_2	p_3	p_4
reservoir q_1	9	21	15	3
both q_2	21	49	35	7
moor q_3	15	35	25	5
no rain q_4	3	7	5	1

Return period of floods: geometric distribution

- Rain amount per $T_d = 10\text{s} = 1\text{wd}$ determined by design: no to minor flooding for $(1, 2, 4)$ & $(8, 9)$, flooding for 18 units r_0.
- Return period T_r of extreme flooding at $t_n = nT_d$ determined by geometric distribution with here $p_n = (1 - p_e)^{n-1} p_e$ where $p_e = P_{24} = q_2 p_4 = 7/256$, s.t.

$$T_r = \mathbb{E}(t_n) = \sum_{n=1}^{\infty} T_d n (1 - p_e)^{n-1} p_e = \frac{T_d}{p_e} \approx 365.7\text{s} \approx 6:06\text{min}.$$

- Standard deviation σ_r (thanks to Daan C & Jason F):

$$\sigma_r^2 = \mathbb{E}\left((t_n - \mathbb{E}(t_n))^2\right) = (1 - p_e)\frac{T_d^2}{p_e^2}$$
$$= (1 - p_e) T_r^2 \Longrightarrow \sigma_r = 36.07\text{wd} = 360.7\text{s} \approx 6\text{min}.$$

Super- and megafloods: geometric distribution of order k

- Two consecutive "2015 Boxing Days" extreme rainfall WEP $p_e^2 = (7/256)^2$ s.t.

$$T_r^{(2)} \approx \frac{T_d}{p_e^2} = (256/7)^2 \times 10\mathrm{s} \approx 223\mathrm{min} \approx 3:43\mathrm{hr}.$$

Movie "Wetropolis Boxing Day flood" on https://github.com/obokhove/wetropolis20162020

- $T_r^{(2)}$ & $\sigma_r^{(2)}$ follow from geometric distribution of order $k = 2$ (Viveros & Balakrishnan 1993, Koutras & Eryilmaz 2017):

$$\frac{T_r^{(k)}}{T_d} = \frac{(1-p_e^k)}{(1-p_e)p_e^k}, \quad \frac{\sigma_r^{(k)}}{T_d} = \frac{\sqrt{1-(2k+1)(1-p_e)p_e^k - p_e^{2k+1}}}{(1-p_e)p_e^k}.$$

Super- and megafloods: Wetropolis-II revisited design

- For floods on two consecutive days with old $p_e = 7/256$:

$$T_r^{(2)} = T_d \frac{(1+p_e)}{p_e^2} = 1374\mathrm{wd} = 13740\mathrm{s} = 3.8\mathrm{hr},$$
$$\sigma_r^{(2)} = T_d \frac{\sqrt{1-5(1-p_e)p_e^3 - p_e^5}}{(1-p_e)p_e^3} = 3.8\mathrm{hr}.$$

- Long waiting times suggest *redesign*, e.g. take Galton board outcome $p_e = p_2 q_2 = 49/256 \approx 1/5$ for $9\mathrm{s}$ rainfall in moor & reservoir, yielding return periods for $k = 2, 3$-day floodings:

$$T_r = 5.2\mathrm{wd} = 52\mathrm{s},\ T_r^{(2)} = 32.5\mathrm{wd} = 5:25\mathrm{min},$$
$$T_r^{(3)} = 175\mathrm{wd} = 29:11\mathrm{min}, \sigma_r^{(k)} \approx T_r^{(k)}, k = 1, 2, 3.$$

Wetropolis-II's' weather: revisited

- X, Q: probabilities p_i rainfall duration/wd versus q_j rain location: https://www.youtube.com/watch?v=g8znktYpxvY
- p_i, q_j with $i, j = 1, 2, 3, 4$ and $\sum p_i = 1, \sum q_j = 1$.
- For current case, $p_1 = q_1 = 3/16, p_2 = q_2 = 7/16, q_3 = p_3 = 5/16, q_4 = p_4 = 1/16$:

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reservoir q_1	9	21	15	3
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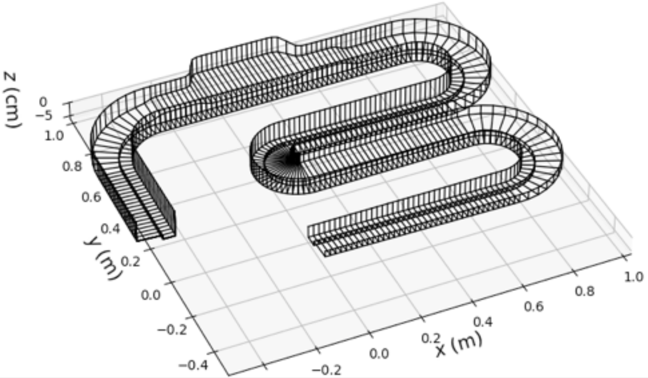
Wetropolis mathematical models

Kinematic river flow in design model fixed wd, r_0; 1D predictive shallow-water river model with ground-water & reservoir dynamics, in progress (PDEs & ODEs: B. et al, HESS, 2020). Bathymetry:



Wetropolis 1D model: maths A, u, h_m, h_{res}, h_{1c}, h_{2c}, h_{3c}

$$\text{River:} \quad \begin{cases} \partial_t A + \partial_s(Au) = S_A \\ \partial_t(Au) + \partial_s\left(Au^2\right) + gA\partial_s h = -g\left(A\partial_s b + \frac{C_m^2 Au|u|}{R^{4/3}}\right) + uS_A \end{cases} \quad \text{on } s \in [0, L]$$

$$\text{with } h = h(A(s,t)), \quad h(s,0) = h_0(s), \quad u(s,0) = u_0(s),$$

$$\text{and } S_A(t) = (1-\gamma)Q_{res}(t)\delta(s - s_{res}) + Q_{moor}(t)\delta(s - s_{moor}) + Q_{1c}(t)\delta(s - s_{1c}) \tag{37a}$$

$$\text{Moor:} \quad \partial_t(w_v h_m) - \alpha g \partial_y\left(w_v h_m \partial_y h_m\right) = \frac{w_v R_{moor}(t)}{m_{por}\sigma_e} \quad \text{on} \quad y \in [0, L_y]$$

$$\text{with } \partial_t h_m|_{y=L_y} = 0, \quad h_m(0,t) = h_{3c}(t), \quad h_m(y,0) = h_{m0}(y) \tag{37b}$$

$$\text{Reservoir:} \quad w_{res}L_{res}\frac{\mathrm{d}h_{res}}{\mathrm{d}t} = w_{res}L_{res}R_{res}(t) - Q_{res}, \text{ with } h_{res}(0) = h_{r0} \tag{37c}$$

$$\text{Canal–1:} \quad w_c(L_{1c} - L_{2c})\frac{\mathrm{d}h_{1c}}{\mathrm{d}t} = Q_{2c} - Q_{1c}, \text{ with } h_{1c}(0) = h_{10} \tag{37d}$$

$$\text{Canal–2:} \quad w_c(L_{2c} - L_{3c})\frac{\mathrm{d}h_{2c}}{\mathrm{d}t} = Q_{3c} - Q_{2c}, \text{ with } h_{2c}(0) = h_{20} \tag{37e}$$

$$\text{Canal–3:} \quad w_c L_{3c}\frac{\mathrm{d}h_{3c}}{\mathrm{d}t} = \gamma Q_{res} - Q_{3c}, \text{ with } h_{3c}(0) = h_{30}, \tag{37f}$$

$$\text{Influxes:} \quad Q_{1c} = C_f\sqrt{g}\, w_c \max(h_{1c} - P_{1w}, 0)^{3/2} \tag{37g}$$

$$Q_{2c} = C_f\sqrt{g}\, w_c \max(h_{2c} - P_{2w}, 0)^{3/2} \tag{37h}$$

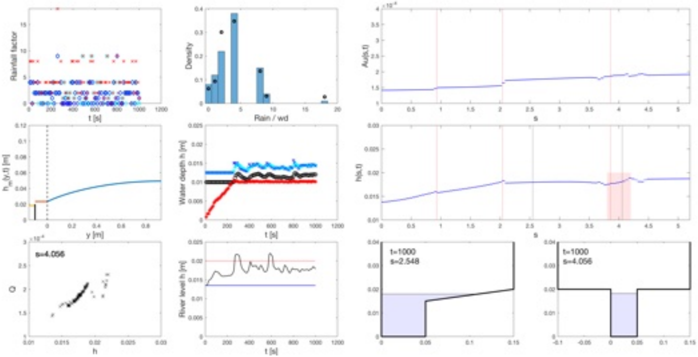
$$Q_{3c} = C_f\sqrt{g}\, w_c \max(h_{3c} - P_{3w}, 0)^{3/2} \tag{37i}$$

$$Q_{moor} = \frac{1}{2} m_{por}\sigma_e w_v \alpha g (\partial_y h_m)^2|_{y=0} \tag{37j}$$

$$Q_{res} = C_f\sqrt{g}\, w_{res} \max(h_{res} - P_{wr}, 0)^{3/2} \tag{37k}$$

Wetropolis modelling

Simulations, `https://github.com/tkent198/hydraulic_wetro`:



Wetropolis flood investigator: future work & proposal

How can a Wetropolis laboratory set-up and a "Numerical Wetropolis Prediction" model be used to understand:

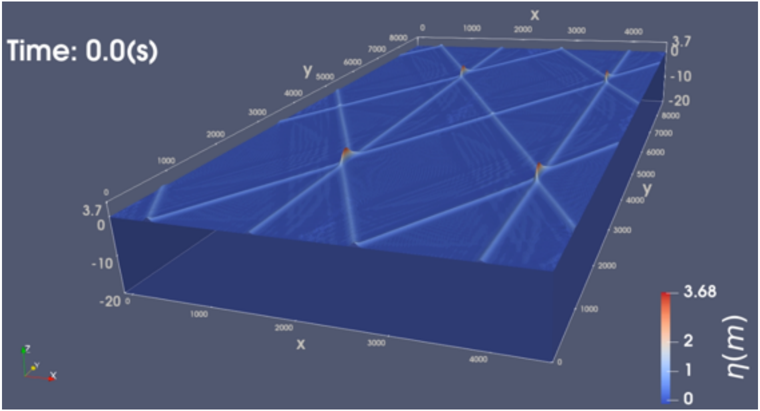
- risk, extreme weather & flooding probability statistics –revisit spatial-temporal rainfall & change-point analysis;
- rare-event simulations (for events of "intermediate rarity");
- flood control –e.g., reservoirs in Wetropolis;
- data assimilation & parameter estimation –experiment as "truth run"; test (limits of) machine learning;
- Wetropolis World's goal: investigate "classical" PDE & Data Assimilation "NWP" model with ML predictions.
- Proposal EPSRC-F^+: PDE/ML, info-gap theory on decision-making, 1/4 educational-version, board game, workshops.

On waves: Modelling extremely high water waves

- Origin 2010 *bore-soliton-splash*:
- Definition rogue/extreme waves: $AI = H_r/H_s > 2$.
- To what extent do exact but idealised extreme- or rogue-wave solutions survive in more realistic settings?
- Will such *extreme waves* fall apart due to dispersion or other mechanisms?
- Use fourfold and ninefold **KPE** amplifications of interacting solitons/cnoidal waves.
- What do you think: will we be able to reach the ninefold wave amplification in more realistic calculations or in reality?

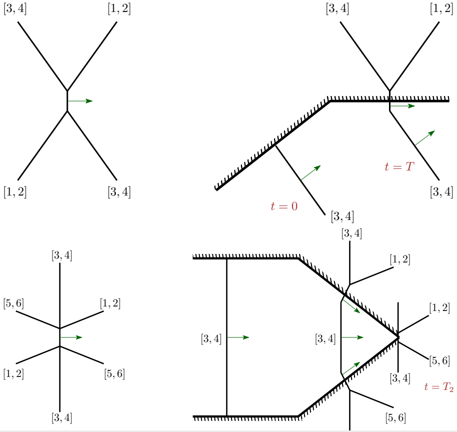
Results simulation three-soliton interaction (dimensional)

Crossing seas (8 domains combined) https://www.youtube.com/watch?v=EGhpQ7BM2jA



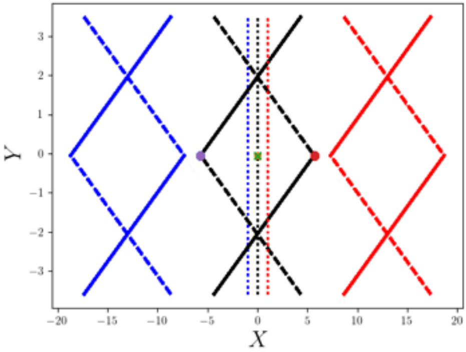
Two & three-soliton interactions in plane vs. wave tank

Left: on infinite horizontal plane; right: top view of wave tank.
Top/bottom: 2 or 3 solitons. (Exact solutions to KP equation.)



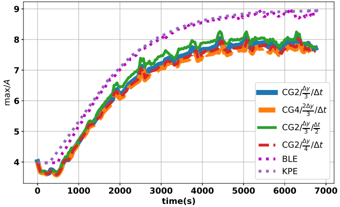
Three-soliton interactions

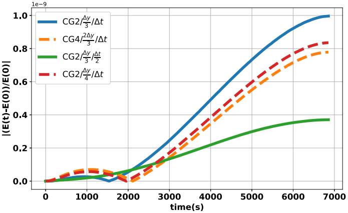
Sketch far-field line solitons at times prior to (blue), at (**black**) & after (red) maximum amplification (geometric & analytical proofs):



Results simulations three-soliton interaction (dimensional)

Kadomtsev-Petviashvili equation (KPE, exact), Benney-Luke equations (BLE), potential-flow equations (PFE, CG-FEM):

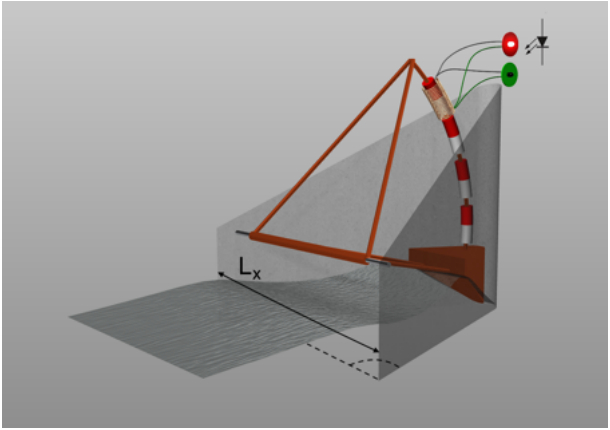




Novel wave-energy device in a breakwater contraction

Proof of principle: https://www.youtube.com/watch?v=SZhe_SOxBWo&t=254s.

Sketch wave amplification in contraction with angle θ_c:



Grand variational principle of novel wave-energy device

Equations of motion follow from variational principle (red=waves, blue=buoy, green=EM-generator, coupling):

$$0 = \delta \int_0^T \int_0^{L_x} \int_{R(t)}^{l_y(x)} \int_0^h -(\partial_t \phi + \frac{1}{2}|\nabla\phi|^2)\mathrm{d}z - gh(\frac{1}{2}h - H_0) - \underline{\frac{1}{2\gamma}\Big(F_+(\gamma(h-h_b)-\lambda)^2 - \lambda^2\Big)}\mathrm{d}y\mathrm{d}x$$

$$MW\dot{Z} - \frac{1}{2}MW^2 - MgZ + (L_i I - \underline{K(Z)})\dot{Q} - \frac{1}{2}L_i I^2 \mathrm{d}t \qquad (1)$$

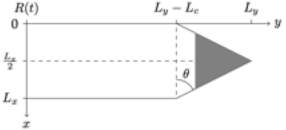
velocity $\mathbf{u} = \nabla\phi(x,y,z,t)$, depth $h(x,y,t)$, rest depth H_0, buoy $h_b(Z,x) = Z - K - \tan\theta(L_y - x)$, piston $R(t)$, coupling function $\gamma_m G(Z) = K'(Z)$, buoy mass M, keel height K, buoy coordinate $Z(t)$, buoy velocity $W(t) = \dot{Z}$, charge $Q(t)$, current $I(t) = \dot{Q}$.

Mathematical modelling: PDEs

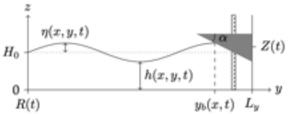
- Potential-flow water-wave dynamics (Laplace equation in interior, kinematic & Bernoulli equations at free surface):

$$
\begin{aligned}
\delta\phi: &\quad \nabla^2\phi = 0 \quad \text{in} \quad \Omega\\
(\delta\phi)|_{z=h}: &\quad \partial_t h + \nabla\phi\cdot\nabla h = \phi_z \quad \text{at} \quad z = h\\
\delta h: &\quad \partial_t\phi + \frac{1}{2}|\nabla\phi|^2 + g(z - H_0) - \lambda = 0 \quad \text{at} \quad z = h.
\end{aligned}
$$

- Coupled elliptic Laplace equation to hyperbolic free-surface equations, plus a (Lagrange) **multiplier** λ.



(b) Top view of the tank and buoy, outlining the tank's dimensions and how the buoy fits the shape of the contraction.



(c) Side view at time t, with the buoy constrained to move vertically.

Mathematical modelling: inequality constraint & ODEs

- Karush-Kuhn-Tucker inequality conditions satisfied at every space-time x, t-position are:

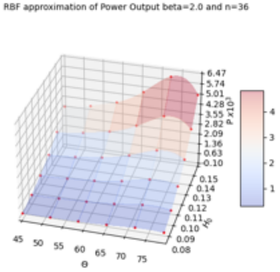
$$\delta\lambda : \lambda = -[\gamma(h-h_b)-\lambda]_+ = -F_+(\gamma(h-h_b)-\lambda)$$
$$\Longrightarrow \underline{h(x,t)-h_b(Z,x)\le 0}, \lambda \le 0, \lambda(h-h_b)=0.$$

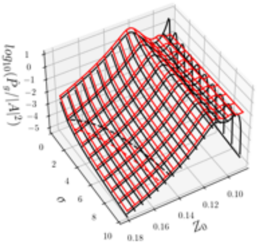
- Add resistance R_i, R_c & Shockley load $V_s(|I|)$ to submodel:

$$\delta W : \dot{Z} = W,$$
$$\delta Z : M\dot{W} = -Mg \underline{-\gamma_m G(Z) I} - \int_0^{L_x}\int_0^{l_y(x)} \lambda \, \mathrm{d}y\mathrm{d}x$$
$$\delta I : \dot{Q} = I,$$
$$\delta Q; \quad L_i \dot{I} = \underline{\gamma_m G(Z)\dot{Z}} - (R_i + R_c) I - \frac{I}{|I|} V_S(|I|).$$

Optimisation wave-energy device

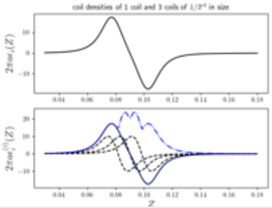
Surrogate modelling geometry angle/rest depth θ_c-H_0 &
1-coil (**black**) vs. 3-coil (red) power P_g (B. et al IEEE2024):

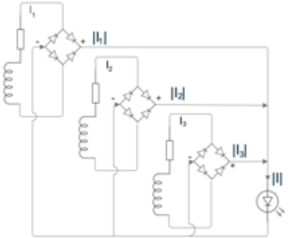




Novel wave-energy device: future work

- **Brief overview given** of wave-energy device, based on extreme-wave amplification in a contraction. Showcased modelling with VPs & geometric numerical integrators.
- Future work: optimisation, real-time control using Pontryagin's principle, surrogate modelling & ML.
- Experiment soliton interactions: `https://www.youtube.com/watch?v=cBhrODnVclU`
- Laboratory realisation under development, for testing of submodels & wave-to-wire models.





Thanks very much for your attention ...

- Knotters, B, Lamb, Poortvliet 2024: How to cope with uncertainty monsters in flood risk management? *Water Prisms*. https://doi.org/10.1017/wat.2024.4 (Nominated paper.)
- B, Kelmanson, Piton, Tacnet 2024: Visualising Flood Frequency, Flood Volume and Mitigation of Extreme Events. https://obokhove.github.io/UKsuccessFEVpreprint23102023.pdf https://www.youtube.com/watch?v=g8znktYpxvY
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