Amplification of interacting solitons in Kadomtsev-Petviashvili and bi-directional water-wave equations

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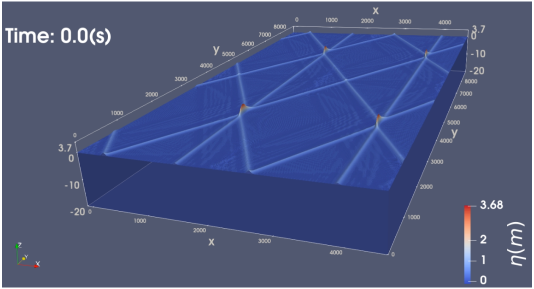
July 6, 2026

School of Mathematics, Leeds Institute for Fluid Dynamics AIMS2026: The Euler water-wave problem



(Lu et al., WW2026 pending.)

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ORIGINAL ARTICLE

Check for updates

Numerical Experiments on Extreme Waves Through Oblique–Soliton Interactions

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Received: 18 February 2022 / Accepted: 14 April 2022 / Published online: 2 June 2022

Abstract
Extreme water-wave motion is investigated analytically and numerically by considering two-soliton and three-soliton interactions on a horizontal plane. We successfully determine numerically that soliton solutions of the unidirectional Kadomtsev–Petviashvili equation (KPE), with equal far-field individual amplitudes, survive reasonably well in the bidirectional and higher-order Benney–Luke equations (BLE). A well-known exact two-soliton solution of the KPE on the infinite horizontal plane is used to seed the BLE at an initial time, and we confirm that the KPE-fourfold amplification approximately persists. More interestingly, a known three-soliton solution of the KPE is analysed further to assess its eight- or ninefold amplification, the latter of which exists in only a special and difficult-to-attain limit. This solution leads to an extreme splash at one point in space and time. Subsequently, we seed the BLE with this three-soliton solution at a suitable initial time to establish the maximum amplification: it is approximately 7.8 for a KPE amplification of 8.4. Herein, the computational domain and solutions are truncated approximately to a fully periodic or half-periodic channel geometry of sufficient size, essentially leading to cnoidal-wave solutions. Moreover, special geometric (finite-element) variational integrators in space and time have been used in order to eradicate artificial numerical damping of, in particular, wave amplitude.

Water Waves (2024) 6:225–277
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ORIGINAL ARTICLE

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A Study of Extreme Water Waves Using a Hierarchy of Models Based on Potential-Flow Theory

Junho Choi[1] · Anna Kalogirou[2] · Yang Lu[3] · Onno Bokhove[3] · Mark Kelmanson[3]

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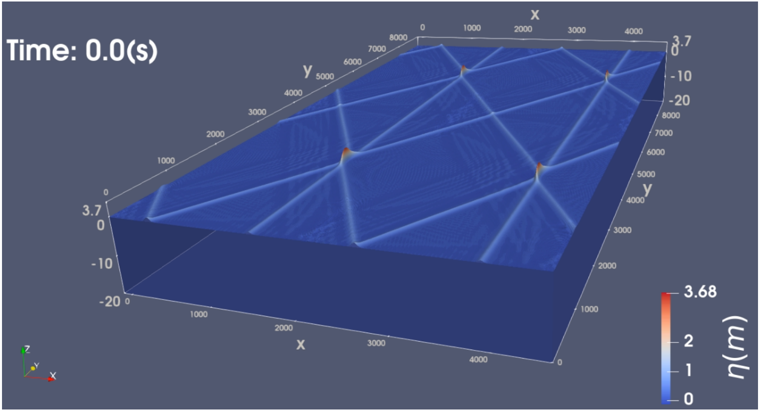
Abstract
The formation of extreme waves arising from the interaction of three line-solitons with equal far-field amplitudes is examined through a hierarchy of water-wave models. The Kadomtsev–Petviashvili equation (KPE) is first used to prove analytically that its exact three-soliton solution has a ninefold maximum amplification that is achieved in the absence of spatial divergence. Reproducing this ninefold maximum paves the way for simulations based on both the Benney–Luke equations (BLE) and more advanced potential-flow equations (PFE). To preserve (for the sake of computations) the region of interaction, exact KPE solutions on an infinite domain are used to yield initial conditions that seed the BLE and PFE models within a periodic domain. The above strategies are realised by directly implementing the corresponding time-discretised variational principles within the finite-element environment *Firedrake*, one aim being automation of the generation of the algebraically cumbersome weak formulations. In the case of three-soliton interactions, it is found numerically that an amplification factor in the interval circa (7.6, 9) can be achieved within the BLE framework, whereas in the PFE framework, this falls to circa 7.8.

Motivation on modelling extremely high water waves

- Origin 2010 *bore-soliton-splash*:
- To what extent do exact but idealised extreme- or rogue-wave solutions survive in more realistic settings?
- Will such extreme waves fall apart due to dispersion or other mechanisms?
- Use fourfold and ninefold KP amplifications of interacting solitons/cnoidal waves.
- What do you think: will we be able to reach the ninefold wave amplification in more realistic calculations, using potential-flow dynamics, or in reality?
- $(4 \times 4)\tilde{A} = 16\tilde{A}$? Laboratory realisations?

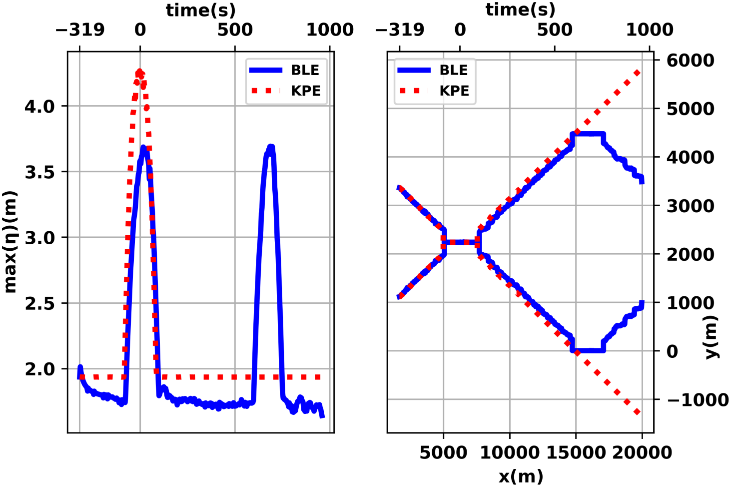
Crossing seas (4 or 8 domains combined –*YouTube*)

`www.youtube.com/watch?v=EGhpQ7BM2jA`



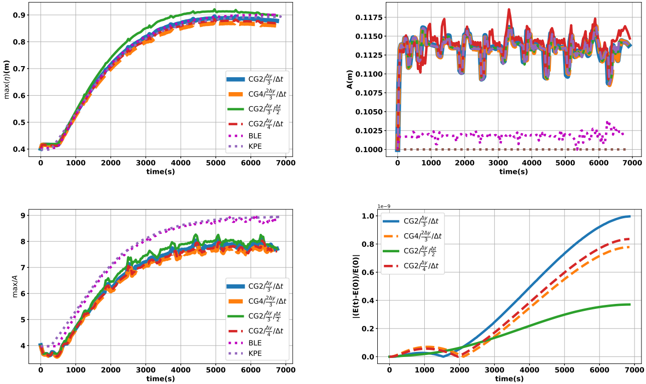
Results simulation three-soliton interaction (dimensional)

Cnoidal waves with periodicity in x, y, t (max. vs. t & x–y tracks):

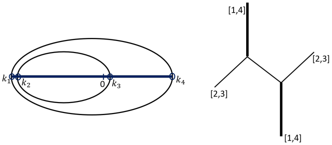


Results BLE- & PFE-simulations three-soliton interactions

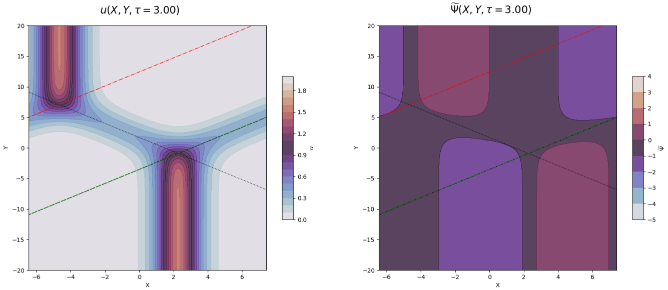
- Demanding PFE simulations: HPC simulation with optimised Additive Schwarz Method-Star pre-conditioner. Amplification 7.5 to 8 at low $\epsilon = 0.01, \delta = 10^{-5}$.



Potential-flow P-type two-soliton/cnoidal interactions

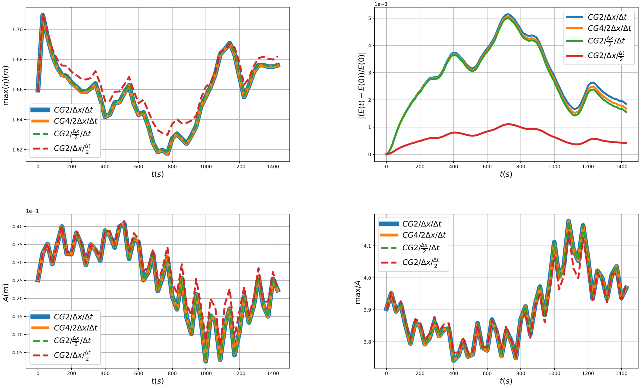


- Sketch (thanks to Prof Yuji Kodama) and exact KPE-solution.
- $K(X,Y,\tau) = (k_3 - k_1)e^{\theta_1}\left(e^{\theta_3} + \frac{k_2-k_1}{(k_3-k_1)}e^{\theta_2}\right) + (k_4 - k_3)e^{\theta_4}\left(e^{\theta_3} + \frac{(k_4-k_2)}{(k_4-k_3)}e^{\theta_2}\right)$, wherein $\theta_i = k_i X + k_i^2 Y - k_i^3 \tau$, $k_1 = -k_4 < k_2 = k_1 + \delta < k_3 = \delta < k_4$, $\delta = 10^{-5}$.



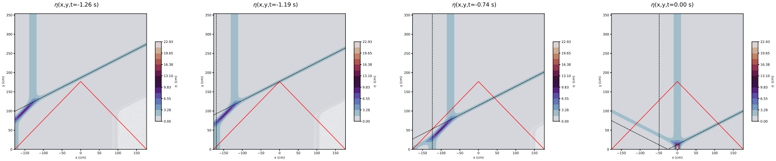
Potential-flow P-type two-soliton interactions

- Demanding PFE simulations with a travelling-wave *P-type web-soliton* with amplitude 4, wavelength 400m, wave height 1.6m, $\epsilon = 0.05$.



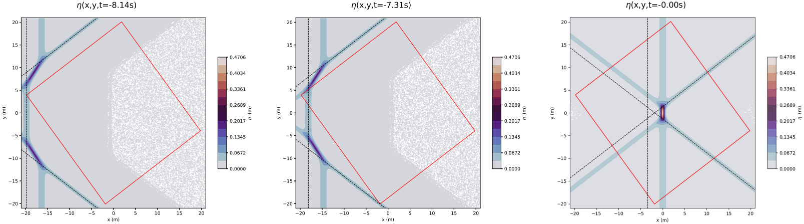
$(4\times 4)\,\tilde{A} = 16\,\tilde{A}$? Laboratory realisations? (Mohammed)

- KP 3-soliton fits in $2\mathrm{m}\times 2\mathrm{m}$ tank imposed by wavemakers at two sides (suggestion V. Shrira –ICTAM2024, Daegu):



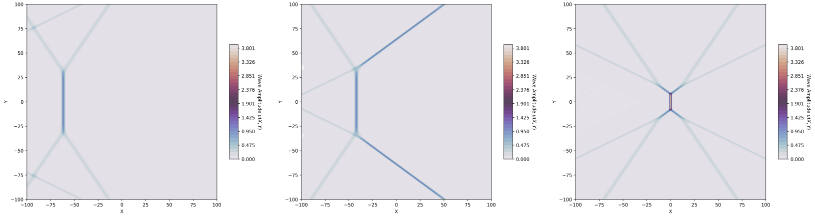
Laboratory realisations?

- KP 3-soliton fits in maritime & $20\mathrm{m} \times 20\mathrm{m}$ tank with wavemakers at two sides:



$(4\times 4)\,\tilde{A} = 16\,\tilde{A}$? By Shoaib Mohammed

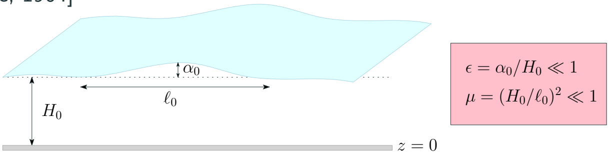
- (Pending) $(4\times 4)\,\tilde{A} = 16\,\tilde{A}$, wavenumbers (guess) $\delta = 10^{-14}, k_5 = -k_4 = \delta, k_6 = -k_3 = 0.7, k_7 = -k_2 = 0.7 + \delta, k_8 = -k_1 = 1.4 - \delta$:



Mathematical hierarchy: PFE, BLE & KPE approximations

⇝ Boussinesq-type approximation: includes weak dispersive effects

- KdV equation: wave propagation in 1D [Korteweg & de Vries, 1895]
- KPE equation: unidirectional propagation in 2DH
  [Kadomtsev & Petviashvili, 1970]
- Benney-Luke equations –BLE: bidirectional propagation in 2DH
  [Benney & Luke, 1964]



Expansion about the sea-bed potential $\Phi(x, y, t) = \phi(x, y, z = 0, t)$, in powers of the small parameter μ [Pego & Quintero, 1999]

- More realistic or parent potential-flow equations (PFE).

Kadomtsev-Petviashvili (KPE) equation

The KPE equation can be obtained from the Benney-Luke equations by introducing the formal perturbation expansions

$$\eta = \tilde{u} + \mathcal{O}(\epsilon^2), \qquad \mathit{\Phi} = \sqrt{\epsilon}\left(\tilde{\Psi} + \mathcal{O}(\epsilon^2)\right),$$

using the transformations

$$X = \sqrt{\frac{\epsilon}{\mu}}\left(\frac{3}{\sqrt{2}}\right)^{1/3}(x-t), \qquad Y = \sqrt{\epsilon}\sqrt{\frac{\epsilon}{\mu}}\left(\frac{3}{\sqrt{2}}\right)^{2/3} y,$$

$$\tau = \epsilon\sqrt{\frac{2\epsilon}{\mu}}\, t, \qquad u = \left(\frac{3}{4}\right)^{1/3}\tilde{u},$$

with $\mu = \epsilon^2$ (in the VP), resulting in the KPE equation in "standard" form

$$\boxed{\partial_X\left(4\partial_\tau u + 6u\partial_X u + \partial_{XXX} u\right) + 3\partial_{YY} u = 0}$$

This equation includes weak effects in the y-direction.

Exact solution of the KP equation

Web and line-soliton solutions can be constructed using Hirota's transformation

$$u(X,Y,\tau) = 2\partial_{XX} \ln K(X,Y,\tau) = \frac{2\partial_{XX}K}{K} - 2\left(\frac{\partial_X K}{K}\right)^2,$$

where function $K(X,Y,\tau)$ can be obtained from the Wronskian

$$K(X,Y,\tau) = \begin{vmatrix} f_1 & f_1^{(1)} & \dots & f_1^{(N-1)} \\ f_2 & f_2^{(1)} & \dots & f_2^{(N-1)} \\ \vdots & \vdots & & \vdots \\ f_N & f_N^{(1)} & \dots & f_N^{(N-1)} \end{vmatrix}.$$

Particular soliton solutions are obtained by taking [Kodama, 2010]

$$f_i = \sum_{j=1}^{M} a_{ij}\, \mathrm{e}^{\theta_j}, \quad \text{where } \theta_j = k_j X + k_j^2 Y - k_j^3 \tau,$$

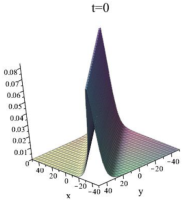
with coefficients k_j being ordered as $k_1 < k_2 < \cdots < k_M$. This solution is called a (N_-, N_+)-soliton, comprising line solitons in the far-field $Y \to \pm\infty$.

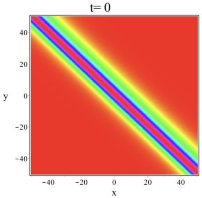
Example: single line soliton

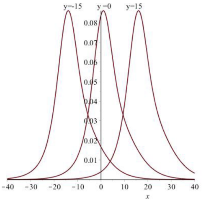
Single line solitons have $(N, M) = (1, 2)$, resulting in $K = f_1 = \mathrm{e}^{\theta_1} + \mathrm{e}^{\theta_2}$ and the line soliton solution is

$$
\begin{aligned}
u(X, Y, \tau) &= \frac{1}{2}(k_1 - k_2)^2 \operatorname{sech}^2 \frac{1}{2}(\theta_1 - \theta_2) \\
&= \frac{1}{2}(k_1 - k_2)^2 \operatorname{sech}^2 \frac{1}{2}\big((k_1 - k_2)X + (k_1^2 - k_2^2)Y - (k_1^3 - k_2^3)\tau\big).
\end{aligned}
$$

The soliton amplitude is $\tilde{A} = \frac{1}{2}(k_1 - k_2)^2$ and its centreline is found by setting the sech^2 argument to zero.







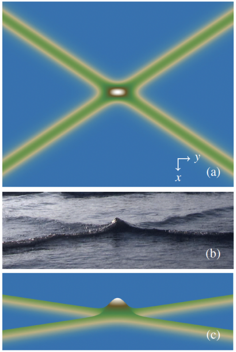
Example: two interacting line solitons

Two line solitons have $(N, M) = (2, 4)$, also called $(2, 2)$-solitons or O-solitons, obtained with functions $f_1 = \mathrm{e}^{\theta_1} + \mathrm{e}^{\theta_2}$, $f_2 = \mathrm{e}^{\theta_3} + \mathrm{e}^{\theta_4}$, and

$$K(X, Y, \tau) = (k_3 - k_1)\mathrm{e}^{\theta_1+\theta_3} + (k_3 - k_2)\mathrm{e}^{\theta_2+\theta_3} + (k_4 - k_1)\mathrm{e}^{\theta_1+\theta_4} + (k_4 - k_2)\mathrm{e}^{\theta_2+\theta_4}.$$

In the far field $Y \to \pm\infty$, we find the single line solitons

$$u_{[1,2]}(X, Y, \tau) = \frac{1}{2}(k_2 - k_1)^2 \mathrm{sech}^2 \frac{1}{2}(\theta_1 - \theta_2 - \ln a),$$
$$u_{[3,4]}(X, Y, \tau) = \frac{1}{2}(k_4 - k_3)^2 \mathrm{sech}^2 \frac{1}{2}(\theta_3 - \theta_4 - \ln b),$$



where a, b depend on k_j. For equal far-field soliton amplitudes $\tilde{A} = \frac{1}{2}(k_2 - k_1)^2 = \frac{1}{2}(k_4 - k_3)^2$, the solution satisfies [Kodama, 2010]

$$2\tilde{A} \leq \max_{(X,Y,\tau)} u(X, Y, \tau) \leq 2\left(1 + \frac{1 - \sqrt{\Delta_o}}{1 + \sqrt{\Delta_o}}\right)\tilde{A},$$

where $0 \leq \Delta_o \leq 1$, hence $\boxed{2\tilde{A} \leq \max u \leq 4\tilde{A}}$.

Example: three interacting line solitons

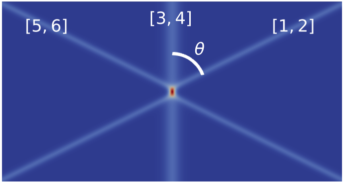
Three line solitons, known as $(3,3)$-solitons, have $(N, M) = (3, 6)$ and functions $f_1 = \mathrm{e}^{\theta_1} + \mathrm{e}^{\theta_2}$, $f_2 = \mathrm{e}^{\theta_3} + \mathrm{e}^{\theta_4}$, $f_3 = \mathrm{e}^{\theta_5} + \mathrm{e}^{\theta_6}$, and

$$\begin{aligned} K(X, Y, \tau) = {} & \underline{A_{135}}\, \mathrm{e}^{\theta_1+\theta_3+\theta_5} + \underline{\underline{A_{235}}}\, \mathrm{e}^{\theta_2+\theta_3+\theta_5} + \underbrace{A_{136}}\, \mathrm{e}^{\theta_1+\theta_3+\theta_6} + A_{236}\, \mathrm{e}^{\theta_2+\theta_3+\theta_6} \\ & + A_{145}\, \mathrm{e}^{\theta_1+\theta_4+\theta_5} + \underline{\underline{A_{245}}}\, \mathrm{e}^{\theta_2+\theta_4+\theta_5} + \underbrace{A_{146}}\, \mathrm{e}^{\theta_1+\theta_4+\theta_6} + \underline{A_{246}}\, \mathrm{e}^{\theta_2+\theta_4+\theta_6}, \end{aligned}$$

with parameter ordering $k_1 < k_2 < k_3 < 0 < k_4 < k_5 < k_6$ & $a, b = 1, c$.

In the far field $Y \to \pm\infty$, we find the single line solitons

$$\begin{aligned} u_{[1,2]} &\approx \frac{1}{2}(k_2 - k_1)^2 \operatorname{sech}^2 \frac{1}{2}(\theta_1 - \theta_2 - \ln \tilde{a}), \\ u_{[5,6]} &\approx \frac{1}{2}(k_6 - k_5)^2 \operatorname{sech}^2 \frac{1}{2}(\theta_5 - \theta_6 - \ln \tilde{b}), \\ u_{[3,4]} &\approx \frac{1}{2}(k_4 - k_3)^2 \operatorname{sech}^2 \frac{1}{2}(\theta_3 - \theta_4), \end{aligned}$$



with $\theta_i - \theta_j = (k_i - k_j)\Big(X + (k_i + k_j)Y - (k_i^2 + k_i k_j + k_j^2)\tau\Big)$.

Example: three interacting line solitons

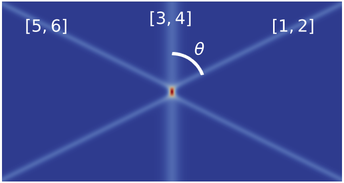
Parameters $k_1, \dots, k_6$ are determined from

$$k_3 + k_4 = 0$$
$$k_5 + k_6 = -(k_1 + k_2) = \tan\theta$$
$$k_4 - k_3 = \sqrt{2\tilde{A}}$$
$$k_6 - k_5 = k_2 - k_1 = \sqrt{2\tilde{A}/\lambda}$$

where angle $\theta > 0$, $\tilde{A} = \frac{1}{2}(k_4 - k_3)^2$ is the amplitude of the $[3, 4]$ soliton, and the outer two solitons are assumed to have amplitude $\tilde{A}/\lambda$, for $\lambda \geq 1$.

Solving the above six equations, gives

$$k_6 = -k_1 = \sqrt{\tilde{A}}\left(\sqrt{2/\lambda} + \sqrt{1/2} + \delta\right)$$
$$k_5 = -k_2 = \sqrt{\tilde{A}}\left(\sqrt{1/2} + \delta\right)$$
$$k_4 = -k_3 = \sqrt{\tilde{A}/2}$$

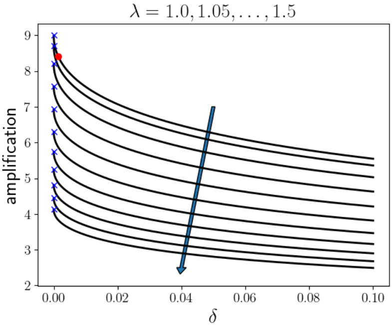


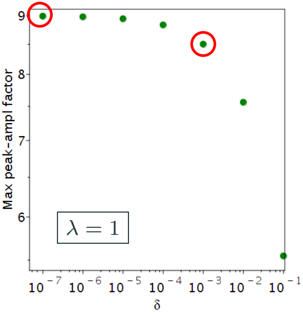
where δ is defined by

$$\delta = \frac{\tan\theta}{2\sqrt{\tilde{A}}} - \left(\sqrt{1/2\lambda} + \sqrt{1/2}\right) > 0.$$

Maximum 9-fold amplification in KP

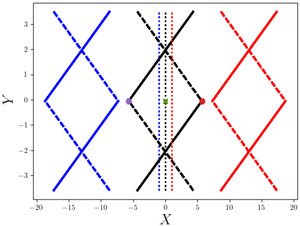
- Proof is based on a geometric argument (additional secondary proof)
- Find 5 centrelines of each of three line solitons (no phase shift at peak)
- Look for intersection points $\rightsquigarrow$ this gives two values of Y, with mean at a unique point $Y_{*\,\delta\to 0} \to -\infty$ when $\tau_* = 0$ and $X_* = 0$
- The space-time point of maximum amplification is (X_*, Y_*, τ_*)
- Amplification: $\dfrac{u(X_*, Y_*, \tau_*)}{\tilde{A}} = \dfrac{(\sqrt{\lambda}+2)^2}{\lambda} + \mathcal{O}(\sqrt{\delta}) \xrightarrow[\delta=0]{} 1 + \dfrac{4}{\lambda} + \dfrac{4}{\sqrt{\lambda}}$

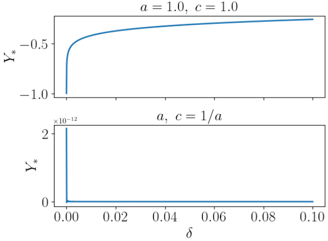




Proof of maximum 9-fold amplification in KP

- Three shift parameters $a, b = 1, c = 1/a$ can be optimised such that splash occurs at $(X^*, Y^*, \tau^*) = (0, 0, 0)$.
- Amplification
  $u(X^*, Y^*, \tau^*)/\tilde{A} = 9 - 8\sqrt{3}\sqrt[4]{2}\sqrt{\delta} + 16\sqrt{2}\delta - 19\, 2^{3/4}\sqrt{3}\delta^{3/2}/3$
- Principle Minor Theorem proves that (X^*, Y^*, τ^*) is a maximum.
- Involved and combined geometrical and analytical proofs (WW2022-2024).





Numerical implementation

Firedrake — *An automated system for the solution of PDEs using the Finite Element Method (FEM).*

Firedrake employs Unified Form Language (UFL) and linear & non-linear solvers PETSc solvers [Rathgeber et al., 2016].

- Space-time discretisation 2nd order of variational principle for BLE: bounded energy oscillations, phase-space conserved.
- Continuous Galerkin (CG) FEM in space for VP, with approximations & test functions/variations $\delta\eta_h,\ \delta\Phi_h$:

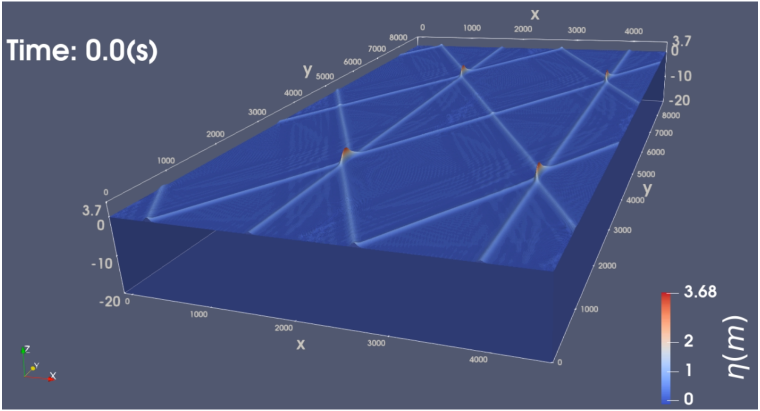
$$\eta(x, y, t) \approx \eta_h(x, y, t) = \sum_k \eta_k(t) w_k(x, y),\ \ldots$$

- Symplectic Störmer-Verlet & MMP time stepping schemes.
- Stable numerical scheme: no artificial amplitude damping . . .

Results simulation three-soliton interaction (dimensional)

Crossing seas (4 or 8 domains combined –*YouTube*)

`www.youtube.com/watch?v=EGhpQ7BM2jA`



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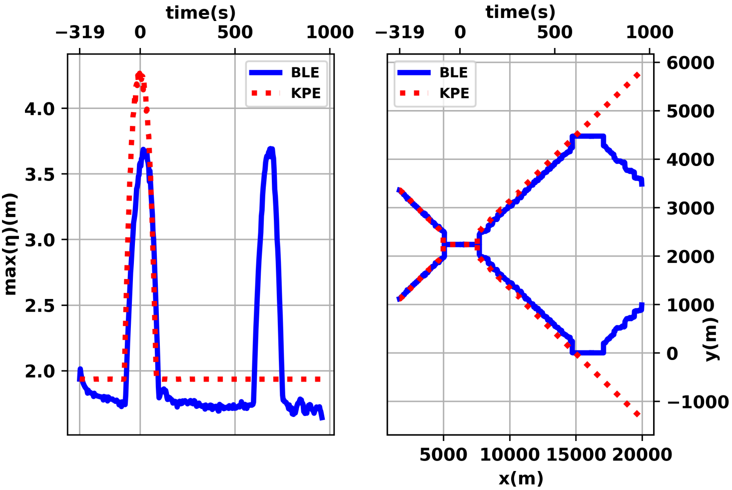
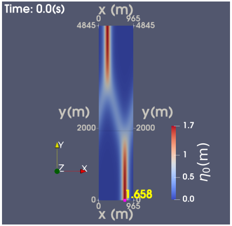
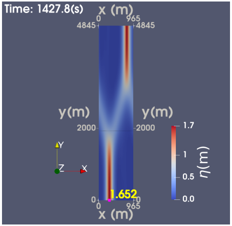


Summary

- 9-fold soliton amplification proven when $\delta \to 0$ & $(X, Y, \tau) = (0.0.0)$
- Web-soliton amplification of KPE is 9^-, seeding BLE-PFE simulations with amplifications ≈ 7.8 & 8.5
- Used novel geometric discretisation of time-discrete VPs, automated via Firedrake, with reduction-of-time-to-development & MPI-HPC.
- How can we reach higher amplitudes; designs wavetank experiments (in progress); $(4 \times 4)\, \tilde{A} = 16\, \tilde{A}$?
- Smoothness computational "periodisation" suboptimal. New P-type web-soliton yields better simulations (Lu et al. 2026):





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O.Bokhove@leeds.ac.uk (EU Eagre: GA859983); GitHub

Discussion

- How realistic is the KPE interaction of $(N \times N)\,\tilde{A} = N^2\,\tilde{A}$ line solitons?
- Are experimental realisations possible?
- Such extreme waves can be used to test maritime engineering objects to scale.

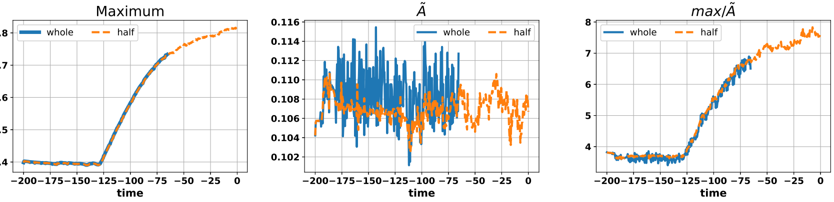
Figure 1.1: Photograph of solitons created at Blackpool Promenade beach, photo taken by Shoaib Mohammed.

Firedrake: exciting aspects & VPs

- Exciting novel & pursued development is to implement (time-discrete) VPs directly via command "*derivative*".
- Advantages: stunning reduction time-to-development.
- New codes more versatile: horizontal mesh with spectral GLL combined with (i) vertical elements with GLL or (ii) 1 vertical element with high-order spectral GLL.
- Firedrake has (automated) MPI-HPC, various preconditioners and also time-integration options.

Computational domain: $\sim$cnoidal waves

- KPE solutions hold on infinite horizontal plane, so domain has to be sufficiently large to eliminate reflection at boundaries.
- Solutions can be set to become approximately periodic in sufficiently large domains.
- Transform $\Phi = U_0(y)x + c_0(y) + \tilde{\Phi}$, where $\tilde{\Phi}$ is periodic, then solve the BLE for η and $\tilde{\Phi}$.
- Doubly or singly periodic domain?



Initial conditions and boundaries

Initial condition consists of two (SP2) or three (SP3) line solitons, expressions of which are known from the KP-solution:

$$\eta_0(x,y) = \eta(x,y,t_0) = 2\left(\frac{4}{3}\right)^{1/3}\partial_{XX}\ln K(X,Y,\tau),$$

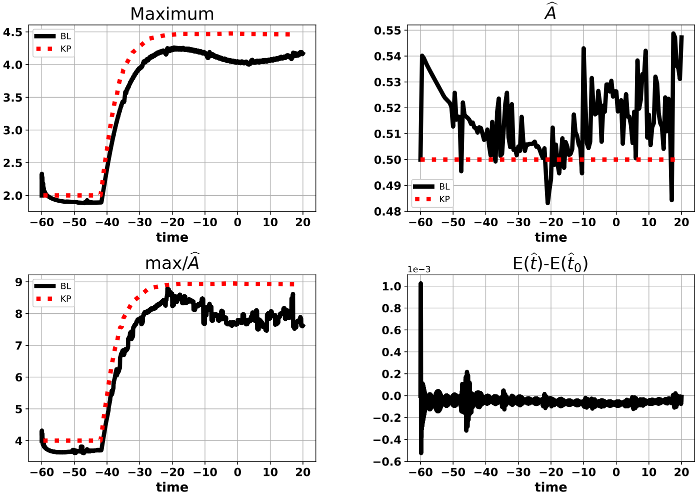
$$\Phi_0(x,y) = \Phi(x,y,t_0) = 2\sqrt{\epsilon}\left(\frac{4\sqrt{2}}{9}\right)^{1/3}\partial_X \ln K(X,Y,\tau).$$

Computational domain is constructed such that initial condition satisfies "periodic boundary conditions" in x–direction.

Case	L_x	L_y	T	N_x	N_y	$\Delta x = \frac{L_x}{N_x}$	$\Delta y = \frac{L_y}{N_y}$	Δt
SP2	10.3	40	50	132	480	0.0779	0.0833	0.005
SP3	20.9	47	200	252	564	0.0829	0.0833	0.005

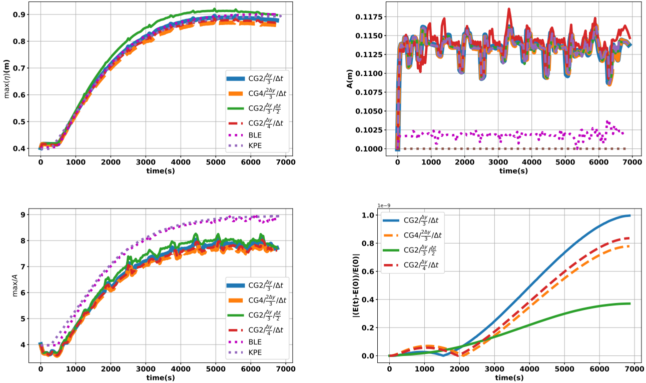
Results BLE-simulations three-soliton interactions

- KPE with $\{\epsilon = 0.05, \delta = 10^{-10}, 9^{-}\times\}$ seeding of BLE simulation yields 7.5 to $8.5\times$ amplification $t_{BLE} \in [-60, 20]$.

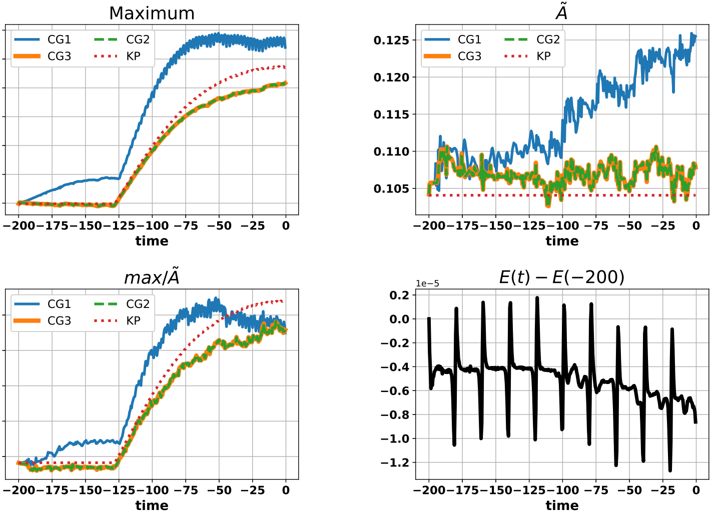


Results BLE- & PFE-simulations three-soliton interactions

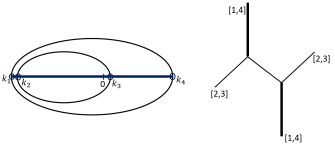
- Demanding PFE simulations: HPC simulation with optimised Additive Schwarz Method-Star pre-conditioner. Amplification 7.5 to 8 at low $\epsilon = 0.01, \delta = 10^{-5}$.



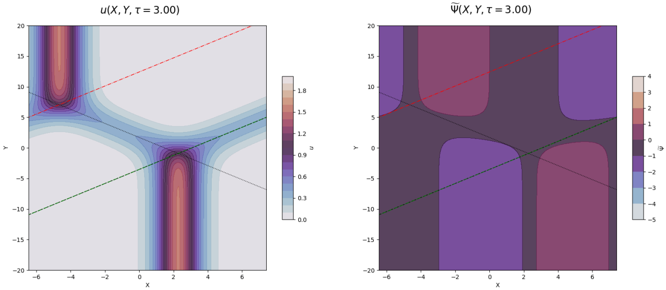
Results BLE-simulation three-soliton interaction



Potential-flow P-type two-soliton/cnoidal interactions

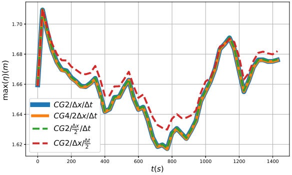


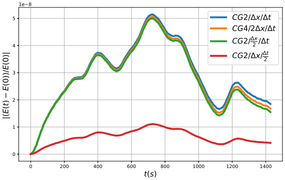
- Sketch (thanks to Prof Yuji Kodama) and exact KPE-solution.
- $K(X,Y,\tau) = (k_3 - k_1)e^{\theta_1}\left(e^{\theta_3} + \frac{k_2-k_1}{(k_3-k_1)}e^{\theta_2}\right) + (k_4 - k_3)e^{\theta_4}\left(e^{\theta_3} + \frac{(k_4-k_2)}{(k_4-k_3)}e^{\theta_2}\right)$, wherein $\theta_i = k_i X + k_i^2 Y - k_i^3\tau$, $k_1 = -k_4 < k_2 = k_1 + \delta < k_3 = \delta < k_4$, $\delta = 10^{-5}$.

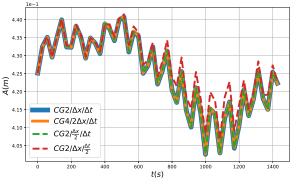


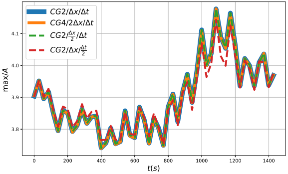
Potential-flow P-type two-soliton interactions

- Demanding PFE simulations with a travelling-wave *P-type web-soliton* with amplitude 4, wavelength $400\mathrm{m}$, wave height $1.6\mathrm{m}$, $\epsilon = 0.05$.



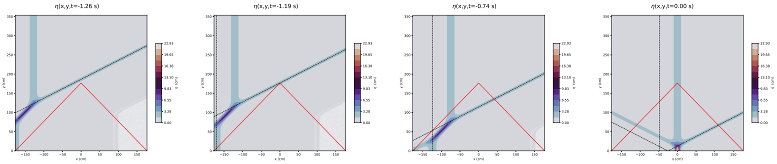






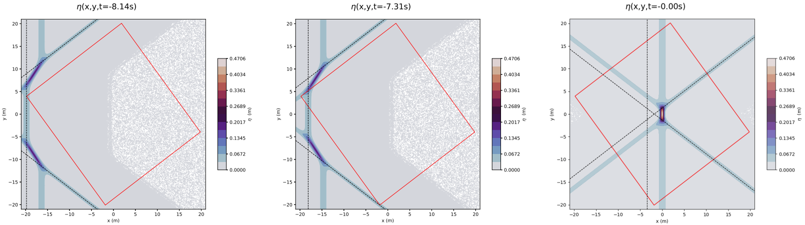
$(4 \times 4)\,A = 16\,A$? Laboratory realisations? (Mohammed)

- KP 3-soliton fits in $2\mathrm{m} \times 2\mathrm{m}$ tank imposed by wavemakers at two sides (suggestion V. Shrira –ICTAM2024, Daegu):



Laboratory realisations?

- KP 3-soliton fits in maritime & $20\mathrm{m} \times 20\mathrm{m}$ tank with wavemakers at two sides:



$(4 \times 4)\,A = 16\,A$? By Shoaib Mohammed

- (Pending) $(4 \times 4)\,A = 16\,A$, wavenumbers (guess) $\delta = 10^{-14}, k_5 = -k_4 = \delta, k_6 = -k_3 = 0.7, k_7 = -k_2 = 0.7 + \delta, k_8 = -k_1 = 1.4 - \delta$: